

## Calculus Solutions: Chapter 4.1

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October 27, 2006

2. The edge of a cube changes from 6 feet to 6.1 feet. Find the actual change in the volume of the cube and then use the local linearization to estimate the change.

**Solution:**

The actual change is

$$(6.1)^3 - 6^3 = 10.981$$

Define the volume function  $V = s^3$  where  $s$  denotes side length. Applying local linearization we approximate the change in the volume to be

$$\Delta V \approx V'(c)\Delta s = 3(6)^2(.1) = 10.8$$

□

4. The radius of a sphere changes from 6 feet to 6.1 feet. Find the actual change in the volume of the sphere and then use the local linearization to estimate the change.

**Solution:**

The actual change is

$$\frac{4}{3}\pi(6.1)^3 - \frac{4}{3}\pi6^3 = 45.9971$$

Define the volume function  $V = \frac{4}{3}\pi r^3$  where  $r$  denotes the radius. Applying local linearization we approximate the change in the volume to be

$$\Delta V \approx V'(c)\Delta r = 4\pi(6)^2(.1) = 45.2389$$

□

6. If the edge of a cube changes by 2%, by about how much does the volume of the cube change?

**Solution:**

$$\Delta V = dV = 3s^2 ds = 3s^2(.02s) = .02(3s^3) = .06V$$

□

8. If a cylinder has base radius 4 feet and the height of the cylinder increases by 3%, how does the volume of the cylinder change?

**Solution:**

$$\Delta V = dV = \pi(4)^2 dh = \pi(4)^2 (.03h) = (.03)(16)\pi h = .03V$$

□

10. Calculate the following, and compare with the approximation given by the local linearization.

b)  $(1.001)^{10}$

**Solution:**

$$(1.001)^{10} = 1.01005$$

Using 4.3

$$(1.001)^{10} \approx 1^{10} + 10(1)^9(.001) = 1.01$$

□

d)  $(1.03)^{-4}$

**Solution:**

$$(1.03)^{-4} = .888487$$

Using 4.3

$$(1.03)^{-4} \approx 1 + -4(1)^{-3}(.03) = .88$$

□

f)  $\sqrt[3]{63}$

**Solution:**

$$\sqrt[3]{63} = 3.97906$$

Using 4.3

$$\sqrt[3]{63} \approx 4 + \frac{1}{3}(4)^{-\frac{2}{3}}(-1) = 3.86772$$

□

h)  $\ln(0.99)$

**Solution:**

$$\ln(0.99) = -0.0100503$$

Using 4.3

$$\ln(0.99) \approx 0 - \frac{1}{.99}(.01) = -0.010101$$

□

l)  $e^{0.03}$

**Solution:**

$$e^{0.03} = 1.03045$$

Using 4.3

$$e^{0.03} \approx 1 + .03 = 1.03$$

□

n)  $e^{-1}$

**Solution:**

$$e^{-1} = 1.10517$$

Using 4.3

$$e^{-1} \approx 1 + .1 = 1.1$$

□

11. Find  $dy$

b)  $y = x^2 + 2x$

**Solution:**

$$dy = (2x + 2)dx$$

□

d)  $y = \sin 4x$

**Solution:**

$$dy = 4 \cos 4x dx$$

□

f)  $y = \sin^2 3x$

**Solution:**

$$dy = 6 \cos 3x \sin 3x dx$$

□

h)  $y = e^{\sin^{-1} x}$

**Solution:**

$$y = \frac{1}{\sqrt{1-x^2}} e^{\sin^{-1} x} dx$$

□