



Mat 132

Practice Problems for the first Midterm

February 26, 2002

The first midterm exam for Math 132 will be held on **Monday, March 4, 8:30-10:00 pm**. The location of the exam depends on your section:

- Sections 1-7 will be in JAVITS 100.
- Sections 8-10 will be in OLD CHEM 116.
- Section 11 will be in JAVITS 103.

You may **NOT** use a calculator. You may **NOT** use any books or notes. Please show up at least 5 minutes early to ensure that everyone has the full 90 minutes to work on the exam. Good luck!

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1. Instructions

The actual midterm will cover sections 5.4-5.7 and 5.10 and 6.1-6.4 of our text. The following problems are representative and are intended to help you study. You should also try to solve all assigned **homework problems**.

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2. Practice Problems

EXERCISE 1. Evaluate the following indefinite integrals:

1. $\int e^{1-x} dx$

2. $\int \tan^4 x \sec^2 x dx$

3. $\int (x-1)^6 (x+2)^2 dx$

4. $\int x \log x dx$

5. $\int \sqrt{1-x^2} dx$

6. $\int \arcsin x dx$

7. $\int \frac{x-1}{x^2-5x+6} dx$

8. $\int \frac{x^2-5x+6}{x} dx$

EXERCISE 2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\int_1^{\infty} e^{-x} \sin x dx$

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$$2. \int_1^{\infty} \frac{e^x}{\sqrt{x}} dx$$

$$3. \int_{-1}^1 \frac{1}{x} dx$$

EXERCISE 3. Area and Volumes:

1. Calculate the area of the region bounded by $\cos x$ and $\sin x$ in the interval $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$.
2. Calculate the volume of the above region (for the same interval as above) when rotated around the y -axis.
3. Calculate the volume of the curve

$$y(x) = x^{-\frac{1}{5}}$$

when rotated around the x -axis for the interval from zero to one.

EXERCISE 4. Find the length of the curve

$$y(x) = \frac{e^{-x} + e^x}{2}$$

for x from -1 to 1 .

EXERCISE 5. Show that: $\int_0^1 x^{1000}(1-x)^{1001} dx = \int_0^1 x^{1001}(1-x)^{1000} dx$

EXERCISE 6. A particle moves along a line. Its velocity is given by the equation

$$v(t) = (t + 1) \sin \pi t.$$

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1. What is the velocity of the particle at $t = 0$, $t = 1$, and $t = 2$?
2. Relative to the starting point, where is the particle at $t = \frac{1}{2}$, $t = 1$, and $t = 2$ (i.e., is the particle: ahead, behind, or at the starting point. **Justify** your answer.)
3. Find the total distance traveled by the particle from $t = 0$ to $t = 2$.
4. Write an expression that represents the average velocity from $t = 0$ to $t = 2$.

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Solutions to Exercises

Exercise 1.

1. Make the substitution $u = 1 - x$.
2. Make the substitution $u = \tan x$
3. Clear first the $(x - 1)^6$ term by making the substitution $u = x - 1$. You will then have to integrate

$$\int (u)^6(u + 3)^2 du.$$

Now expand the binomial $(u + 3)^2$ term and distribute the u^6 term to each summand. Integrate each term of the sum.

4. Use integration by parts

$$\int x \log x dx = \frac{x^2}{2} \log x - \int \frac{x}{2} dx$$

5. Use a trigonometric substitution like $x = \sin u$, or $x = \cos u$
6. Use integration by parts

$$\int \arcsin x dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$$

7. Factor the denominator. Decompose the fraction into a sum of simple fractions and then integrate each summand.
8. Divide over the denominator. You should obtain a sum of three terms each of which is easy to integrate.

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Exercise 2.

1. To see that the integral is convergent you may use the comparison test. Show that

$$e^{-x} \sin(x) \leq e^{-x}$$

for all x , and prove that the improper integral

$$\int_1^{\infty} e^{-x} dx$$

is convergent.

Alternatively one may show convergence and compute the integral directly using twice integration by parts:

$$\begin{aligned} \int_1^t e^{-x} \sin x dx &= -e^{-x} \sin x \Big|_1^t + \int_1^t e^{-x} \cos x dx & f' = e^{-x}, g = \sin x \\ &= -e^{-x} \sin x \Big|_1^t - e^{-x} \cos x \Big|_1^t - \int_1^t e^{-x} \sin x dx \end{aligned}$$

which yields

$$\int_1^t e^{-x} \sin x dx = \frac{1}{2}(-e^{-x} \sin x \Big|_1^t - e^{-x} \cos x \Big|_1^t)$$

Since

$$\lim_{t \rightarrow \infty} e^{-x} \sin x = \lim_{t \rightarrow \infty} e^{-x} \cos x = 0$$

it follows that the improper integral is convergent and has value

$$\int_1^{\infty} e^{-x} \sin x = \frac{1}{2e}(\sin(1) + \cos(1))$$

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2. Compare to $e^{\frac{x}{2}}$, or to x^2 . The integral is divergent.
3. Be Careful! This is an actually an improper integral since the function is not defined at zero. Divide the interval into two pieces and discuss each improper integral separately. **Note:** The answer is not zero.

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Exercise 3.

1. Sketch the graphs of $\sin(x)$ and $\cos(x)$ and/or observe that

$$\sin(x) \geq \cos(x), \quad \frac{\pi}{4} \leq x \leq \frac{5\pi}{4}.$$

The area is equal to

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx.$$

2. Use the 'washer' (shell) method. The volume is then given by the following integral:

$$2\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} x(\sin x - \cos x) dx$$

which can be evaluated using integration by parts.

3. Use the 'disc' method. The volume is then given by $\pi \int_0^1 (x^{-\frac{1}{5}})^2 dx$. This is an improper integral. Check to see that the integral converges. The volume is $\frac{5\pi}{3}$.

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Exercise 4. Calculate $\frac{dy}{dx}$ and use the formula for the length of a curve:

$$\begin{aligned}\text{Length} &= \int_{-1}^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_{-1}^1 \sqrt{1 + \left(\frac{-e^{-x} + e^x}{2}\right)^2} dx \\ &= \int_{-1}^1 \sqrt{\left(\frac{e^{-x} + e^x}{2}\right)^2} dx = \int_{-1}^1 \frac{e^{-x} + e^x}{2} dx\end{aligned}$$

and now the last integral is easy to evaluate...

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Exercise 5. Use the substitution $u = 1 - x$. Then $du = -dx$. Find the new limits of integration after substitution. Recall also that we have $\int_a^b f = -\int_b^a f$. Exercise 5

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Exercise 6.

1. $v(0) = v(1) = v(2) = 0$
2. For $t = \frac{1}{2}$ and $t = 1$ the answer is clear: the particle is ahead of the starting position. Indeed, the function $v(t)$ is non-negative in the interval from $t = 0$ to $t = 1$. Hence the displacement of the particle, i.e., the integral of the velocity function, is positive.

Sketch the graph of the velocity function. Now observe that if the function were simply $\sin \pi t$ then at each integer the particle would be back at the starting point. And the displacement at each even integer would be zero. However, $t + 1$ is an increasing function. Hence the area swept out by $(t + 1) \sin(\pi t)$ between consecutive integers is increasing. Thus the particle is behind the starting point at even integers (a negative displacement). More precisely we have,

3. Total distance equals

$$\int_0^2 |v(t)| dt = \int_0^1 v(t) dt - \int_1^2 v(t) dt$$

4. This is the average value of the function $v(t)$. Which is: $\frac{1}{2-0} \int_0^2 (t+1) \sin \pi t dt$

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