

Problem Set 6 : Elementary Counting

Due 03/09/04

As usual, think about all problems, and come up with some ideas about how to solve them. Then choose one or two of the problems and write up a careful solution.

For the following problems you need to know these facts:

- The number of permutations of n different objects is: $n! = n(n-1)(n-2)\dots 2 \cdot 1$, with the convention $0! = 1$.

- If we have n_1 identical objects of type 1, n_2 identical objects of type 2, \dots and finally n_k identical objects of type k , the number of permutations of all these $n_1 + n_2 + \dots + n_k$ objects is

$$\frac{(n_1 + n_2 + \dots + n_k)!}{n_1!n_2!\dots n_k!}.$$

- The number of different ways to choose k objects from a set of n objects is:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

1. Assume that n_1, n_2, \dots, n_k are given as above, and $n = n_1 + n_2 + \dots + n_k$.

(a) Show that:

$$\frac{(n_1 + n_2 + \dots + n_k)!}{n_1! \cdot n_2! \cdot \dots \cdot n_k!} = \binom{n}{n_1} \binom{n-n_1}{n_2} \cdot \dots \cdot \binom{n-n_1-n_2-\dots-n_{k-1}}{n_k}$$

(b) Explain why the right-hand side represents the same number of permutations.

2. In Deep Blue Chess Club there are 2 girls and 7 boys. A team of 4 players is to be chosen for a tournament, and there must be at least a girl on the team. In how many different ways can they choose their team?

3. Count the number of ways one can group 48 distinct people into 24 pairs.

4. A soccer team has 11 players.

(a) How many different ways are there to select a captain and a deputy captain?

(b) How many different ways are there to select two co-captains?

(c) How many different ways are there to select an order for the players to collect their gold medals?

5. A poker hand consists of five cards from an ordinary deck of cards. Why is "three-of-a-kind" a higher ranked poker hand than "two-pair"?

6. We roll a fair six-sided die exactly three times. Determine the probability that the sum of the values rolled equals eleven.

7. You hand a friend a standard deck of 52 playing cards face down. You ask him to divide the deck into three sub-decks, using simple cuts, and to place them face down on the table. Then you say "I'll bet you even money that one of those three top cards is a face card" (a jack, queen, or king). Would your friend be wise to accept the bet?