

## Problem Set 5 : Counting

Due 03/02/04

As usual, think about all problems, and come up with some ideas about how to solve them. Then choose two of the problems and write up a careful solution.

For the following problems you need to know these facts:

### Permutations

Let's consider in how many ways you can arrange for 10 people to speak at a conference. Any of them can talk first, so we have 10 choices. Any of the remaining 9 can give the second talk then you have 9 choices, for the third speaker there are 8 choices, and so on till the last person. So the number of arrangements is :  $10 \cdot 9 \cdot 8 \cdot \dots \cdot 1$

**General Rule** The number of permutations of  $n$  different objects is

$$n! = n(n-1)(n-2) \cdots 2 \cdot 1$$

with the convention  $0! = 1$ .

### Permutations with repetitions

If we have  $n_1$  identical objects of type 1,  $n_2$  identical objects of type 2,  $\dots$  and finally  $n_k$  identical objects of type  $k$ , the number of permutations of all these  $n_1 + n_2 + \dots + n_k$  objects is

$$\frac{(n_1 + n_2 + \dots + n_k)!}{n_1! n_2! \cdots n_k!}.$$

Take a moment to think why this is true.

*Example:* Having 2 red, 3 green and 5 blue balls, there are  $(2 + 3 + 5)! / (2!3!5!) = 2520$  different ways to arrange them in a row.

1. In how many ways can you arrange 10 people to sit around a round table. Note that different ways obtained by rotating them around the table will be considered the same.
2. The Hermitian language has only three letters : A, B and C . A word in this language is any sequence of letters (repetitions allowed).
  - (a) How many four-letter words are there in this language ?
  - (b) How many words are there which are fewer than ten letters long ?
  - (c) How many of the nine-letter words start with A ?
3. How many 10 digit numbers have at least 2 digits equal?
4. For each of the following examples, find the number of different words you can obtain by permuting the letters. (A word doesn't have to make sense.)

“LOST”

“APPLE”

“CHEESE”

“APPLESAUCE”

5. Simplify the following expressions:

$$\frac{10!}{7!3!} \quad \frac{n!}{(n-1)!} \quad \frac{(n+1)!}{n(n+1)}$$

6. On a  $5 \times 8$  board, a rook is placed on the lower left corner of the board. Assume that the rook can only move horizontally to right or vertically upward. In how many different ways, it can reach the upper right corner of the board? (*Hint:* Every possible path corresponds to a word having eight R's, when it is moving a unit rightward, and five U's, when it is moving a unit upward, and every such word determines a path. For example, the word RUURRRURRUURR corresponds to the path determined by moving a unit right, two units up, three units right, a unit up, two units right, two units up and finally two units right to get to the upper right corner.)