

Problem Set 12

Due 04/27/04

As usual, think about all problems, and come up with some ideas about how to solve them.

1. All the students in a school are arranged in a rectangular array. The tallest student in each row is identified, and among these students John is the shortest. Then in each column the shortest student is identified, and Mary is the tallest of these. Who is taller: John or Mary ?
2. A 3x3 table is filled with numbers. It is allowed to increase each number in any 2x2 square by 1. Is it possible, using these operations, to obtain the table shown below starting with a table filled with zeros ?

$$\begin{bmatrix} 4 & 9 & 5 \\ 10 & 18 & 12 \\ 6 & 13 & 7 \end{bmatrix}$$

3. The entire plane is colored using two colors. Prove that there are two identically colored points exactly 1 foot apart.
4. Two towns A and B are on the same side of a river. A water pump station S is to be built somewhere by the river and be connected by straight pipelines to A and B . Where should S be built in order to minimize the total length of the pipelines ?
5. A bug is at the vertex of a wooden cube of side length 1 inch and wants to reach the opposite vertex (the only vertex which does not lie on the same face as the starting point). Which path makes the trip shortest, and what is its length ?
6. On all parallelograms with a given perimeter, which has the greatest area ?
7. Prove the Pythagorean theorem. (Hint: you are allowed to use a text-book).