

Problem Set 11 : Games

Due 04/20/04

As usual, think about all problems, and come up with some ideas about how to solve them.

In each problem, there is a game in which two players take turns making moves, and a player cannot decline to move. The problem is always to find out which player (if any) has a winning strategy. *Try playing the games first.* Start with simpler cases and try to find a pattern by understanding each simple case completely.

Recall that a *winning strategy* is a strategy that one player can follow to guarantee that she/he will win.

1. Remember the game we discussed in class. Two players start with a pile of 27 stones. In each turn, a player can remove one or two stones from the pile. The one who removes the last stone will be the winner of the game. Who has a strategy for winning and what is that?
2. Two players take turns putting pennies on a perfectly round, symmetrical table, one penny per turn, without piling one penny on top of the other. The player who cannot place a penny loses. Who can win and how?
3. There are two piles each containing 7 pennies. At each turn, a player may take as many pennies as she chooses (but at least one) from one of the piles (either pile). A player loses if she cannot move. Who can win and how?
4. Same as previous game, but this time, if a player can't move this time she *wins*.
5. The game starts with number 1 written on the blackboard. The players take turns adding any integer from 1 through 9 to the current number. The player who reaches the number 100 wins. Who can win and how?
6. Consider the previous game, but this time the player who reaches the number 100 (or a larger number) loses.