

Problem Set 10 : Games

Due 04/13/04

As usual, think about all problems, and come up with some ideas about how to solve them.

In each problem, there is a game in which two players take turns making moves, and a player cannot decline to move. The problem is always to find out which player (if any) has a winning strategy. *Try playing the games first.* Start with simpler cases and try to find a pattern by understanding each simple case completely.

Recall that a *winning strategy* is a strategy that one player can follow to guarantee that she/he will win.

1. There is a pile of 37 pennies on a table. The first player divides the pile into two smaller piles (not necessarily equal). The second player chooses a pile and divides it into two smaller piles. They continue. Each turn a player chooses one of the piles on the table and divides it into two smaller piles. The player who cannot do this anymore is the loser. Who wins and how?
2. A rook stands at the bottom left corner of a chessboard. Players take turns moving the rook as many squares as they want, but it can be either horizontally to the right or vertically upward. The player who places the rook on the top right corner square wins. Who can win and how?
3. Same as the last game but with a king and a king's moves. It starts on the bottom left corner again. Each time a player can move it either one square to the right, or one square upward, or diagonally one upward and one to the right. Again the player who places the king on the top right corner square wins. Who can win and how?
4. A general needs to take his troops across the river. He spies two boys with a small boat. He commandeers both the boat and the boys. Unfortunately, the boat will only hold two boys or one soldier. Yet he determines a method for getting his troops across. What could be? If there are n soldiers, how many times is the boat going to cross the river in order for everybody to have crossed the river?
5. In a distant land, bigamy is common. There are six people who want to cross a river in this land. This group consists of two men, each with two wives. No man can tolerate any of his wives being in the company of another man unless he is present to chaperon. There is a boat that holds two people to be used for crossing the river. Can they do it? In how many crossings?
6. Same problem as above except the boat will hold three people.