Homework 11 MAT 515

Prove the following statements (or answer, if appropriate). Hand in Problems 2, 7, 8, 10, 13, 14(b), 14(e).

(1) An isometry maps segment to congruent segments, angles to congruent angles, circles to congruent circles.
(2) A reflection about a line is an isometry.
(3) A symmetry about a point is an isometry. (Can you prove it without using the fact that a symmetry about a point is a special kind of rotation?)
(4) A rotation about a point is an isometry.
(5) Each point in the plane is uniquely determined by its distance to three non-collinear points. (Thus, if P and P' are points, and $PA \cong PA'$, $PB \cong PB'$ and $PC \cong PC'$ then $P = P'$.)
(6) Use the previous statement to give a proof (different from the one in class) that an isometry is determined by three non-collinear points and their images.
(7) The composition of two translations is translation and that for each triple of points A, B, C in the plane, $T_{BC} \circ T_{AB} = T_{AC}$. Also, two translations commute, that is, for each four points in the plane, $T_{AB} \circ T_{CD} = T_{CD} \circ T_{AB}$.
(8) Does the composition of two symmetries about points commute? What about two reflections about two lines? Two rotations? (Recall that two transformations S and T commute if $S \circ T = T \circ S$.)
(9) A translation maps a line segment to a parallel line segment.
(10) If α is an angle different from 0, then the only fixed point of a rotation about point P through α is P.
(11) The set of fixed points of a reflection about a line is the line.
(12) If a function from the plane to the plane preserves distances then it is bijective.
(13) Prove that the composition of $S_A \circ S_A$ of symmetry about a point A with itself is the identity. Deduce that the inverse of a symmetry about a point is the same symmetry about the same point. Formulate and prove analogous statements about reflections.
(14) Find all the symmetries of the following figures: (that is, all the isometries that leave each of the figures invariant. A figure is invariant under an isometry if the image of the figure by the isometry coincides with the figure, not necessarily pointwise.)
   (a) An isosceles triangle.
   (b) A rectangle.
   (c) A square.
   (d) A circle.
   (e) A regular n-sided polygo
(15) We studied five types of isometries: translations, rotations, symmetries about a point, reflections about a line and glide reflections. Make a table as we did in class, studying what happens when one compose two isometries, one of each of the five types. You need to consider twenty five cases, and some have sub cases!