Background:

f: M compact, connected manifold $\partial M = \phi$

$f: M \to S^p$ (y a regular value for f)

Thm. A: If y, y' are regular values of f and $\mathcal{B}, \mathcal{B}'$ are +- oriented bases for TS^p_{*} ($*$ ↑ y or y')

$(f^{-1}(y), f^{*}\mathcal{B})$ is frame cobodant with $(f^{-1}(y'), f^{*}\mathcal{B}')$

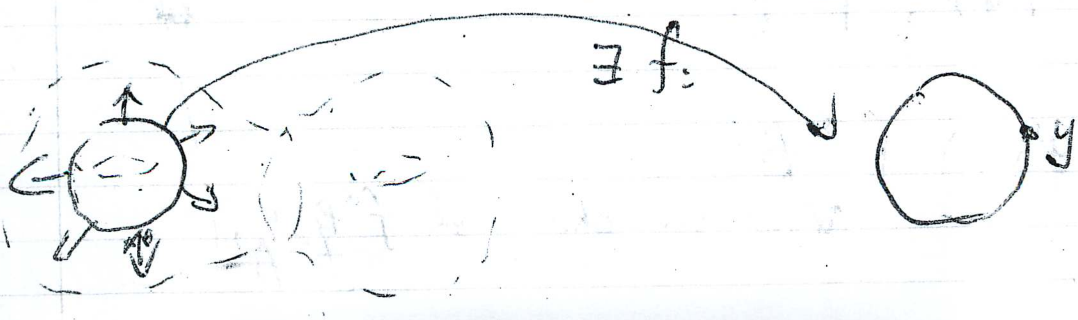
Thm B: $f: M \to S^p$, $g: M \to S^p$

smoothly homotopic

$$\Longleftrightarrow (f^{-1}(y), f^{*}\mathcal{B}) \underset{\text{Cobodant}}{\overset{\text{Frame}}{\sim}} (g^{-1}(y), g^{*}\mathcal{B})$$

Thm C: Any compact framed submanifold $(N, \mathcal{B}')$ of codimension p in $\underset{m}{M}$ $n+p=m$

occurs as a pontryagin manifold for some $f: M \to S^p$

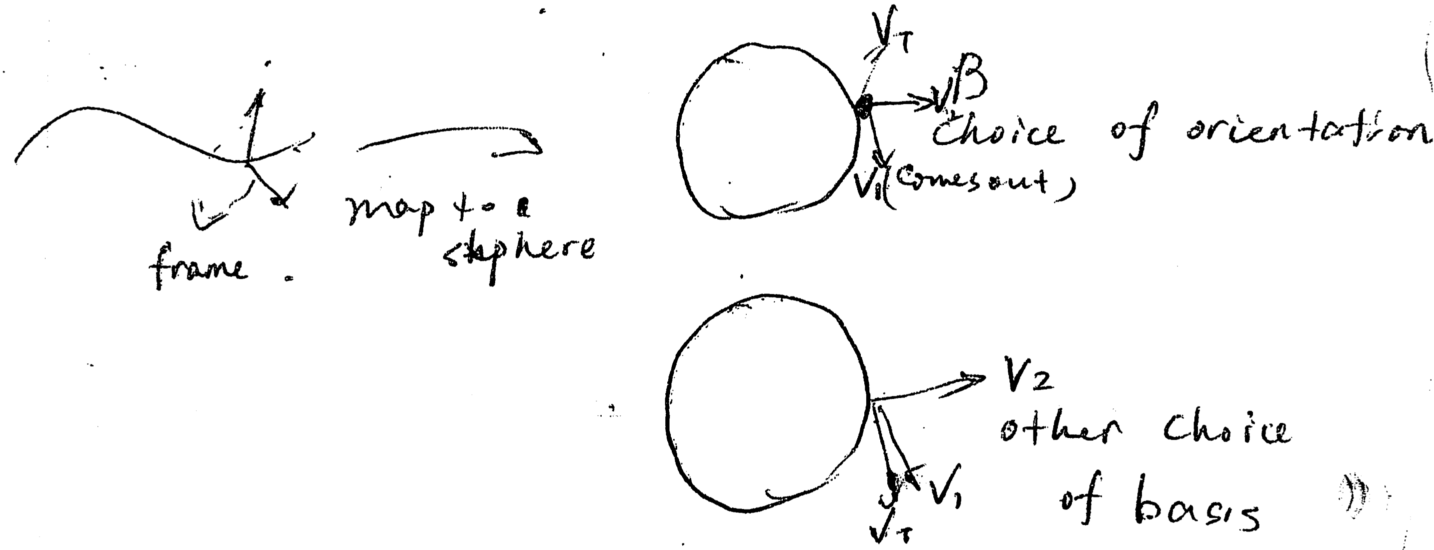


on a sphere.

Outline proof of $\underline{A}$

① show $(f^{-1}(y), f^{*}B) \overset{EC}{\sim} (f^{-1}(y), f^{*}B)$

What does this mean?



Now we want transform the basis at lower sphere to the upper one.

And ∃ there's a linear map

∃ linear map with positive determinant

$G: (x, y, V^T) \to (x', y, V^T)$

& space all such g $P \times P$ matrix with positive determinant.

$$\boxed{\begin{array}{l} (f^{-1}(y), f^{*}B) \\ \hookrightarrow (f^{-1}(y'), f^{*}B') \\ \text{Now we changed } f^{*}B \text{ to } f^{*}B' \end{array}}$$

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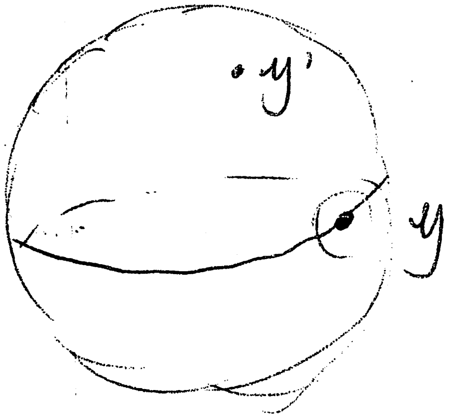
2.

Your coments follow
draft pages



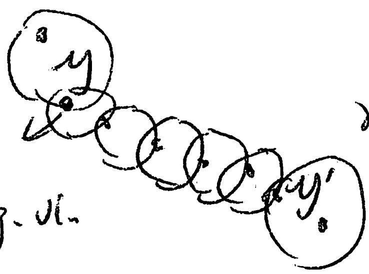
If y' is close to y

Since f has islated critical values

Can find "a little" neighborhood of y

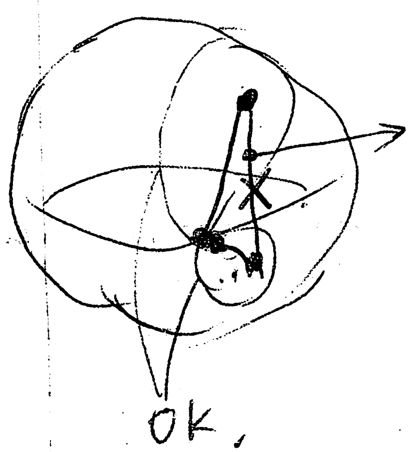
so that $\forall z \in N_\varepsilon(y)$, z is a regular value of f.

$\therefore f^{-1}(y) \sim f^{-1}(z)$ homotopy by rotation of the sphere.



z is a reg. vl.

make a path, avoid the critical points. Since critical points are isolated, so it can be done.



Hopf Thm $\dim M = n$, if M connected, orientable without boundary $\partial M = \emptyset$

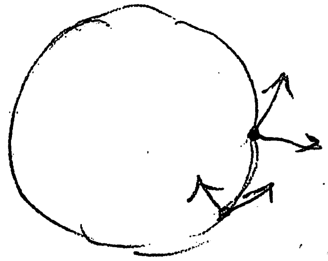
$f: M^n \to S^n$ $\quad g: M^n \to S^n$

f & g are smoothly homotopic $\Leftrightarrow \deg f = \deg g$.

If M is non-orientable

$f \overset{\text{Smooth}}{\sim} g \Leftrightarrow \deg f = \deg g \mod 2$

M nonorient:



need to go around even times to change the degree.

S^0 (a pair of points the framing is nothing, and get homotopy)

~~manifold~~ its

$N^n \subset M^m$

$m - n = p$

Duality. { sub manifold of Dim n & Dim p are related $\to$ ST.

" Understand something by understanding its "dual" "

e.x.