XINYU

12/8/11

Consider Hopf fibration of $S^3$

Cobodism

$N_1, N_2$ are cobordant if

$N_1 \times [0, \varepsilon]$, $N_2 \times [1-\varepsilon, 1]$

Fit ~~insight~~ inside $M \times [0,1]$ with

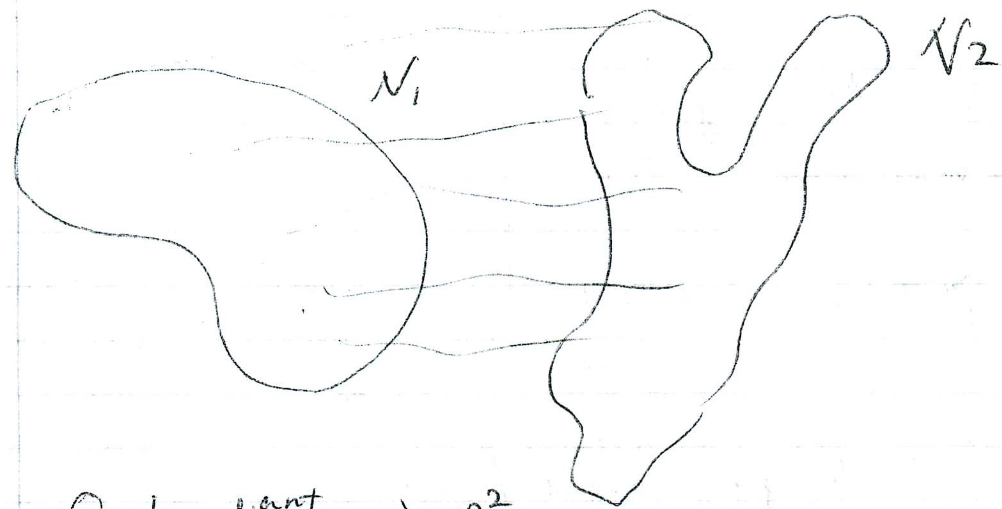
$M \times \{0\} = N$, $M \times \{1\} = N_2$

$X \subset M \times [0,1]$ with $\partial X = N_1^{\times\{0\}} \cup N_2 \times \{1\}$

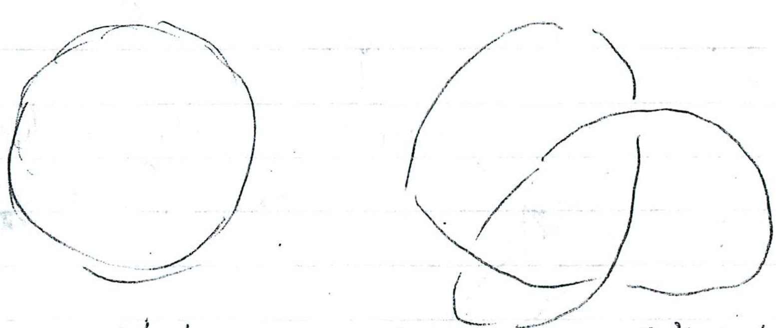
Framing for $N^n \subset \cancel{\mathbb{R}^m}\, M^m$ $m > n$

Choose a positive basis for $TN$.

Extend to basis for $TN^{\perp}$




Cobord~~ism~~ant in $\mathbb{R}^2$

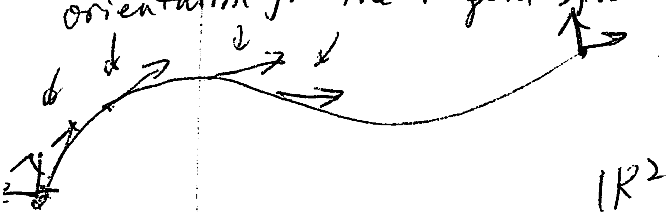


Not Cobordant in $\mathbb{R}^3$ (but is in $\mathbb{R}^4$)

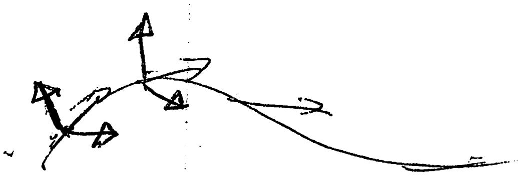
Framing:

"Choice of orientation for what's left over"

orientation for the tangent space



$\mathbb{R}^2$



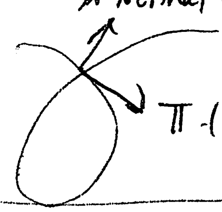
$\mathbb{R}^3$

but need to choose two vectors at each place.

In Calculus III.

take $\mathbb{T}$, $\mathbb{N}$, $\dfrac{\mathbb{T} \times \mathbb{N}}{\|\mathbb{T} \times \mathbb{N}\|}$

$\mathbb{N}$ Normal vector



$\mathbb{T}$ (tangent vector)

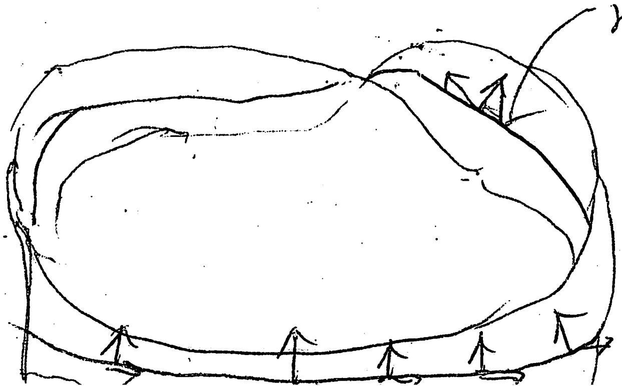
"You can't always choose frames"

Not all submanifolds have a framing.

Mobius strip $= M$

$\mathbb{N}$

midline doesn't have the framing



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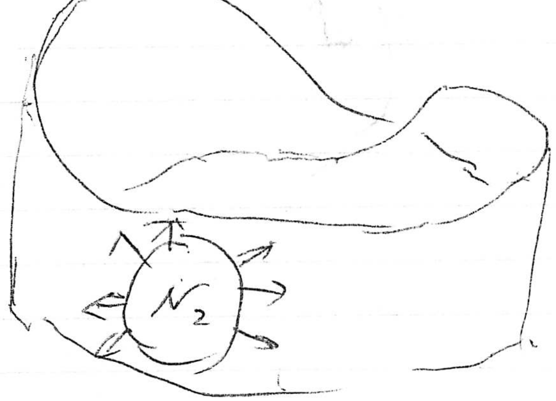
Mobius strip $= M$ Midline $= N$.
can't make a framing for $N \subset M$.

because of the fact that $M$ is not orientable

However:




$N_2$ is fine.

(Note if $M$ is "Fat Mobius" $= M \times [-\varepsilon, \varepsilon]$, ok again)

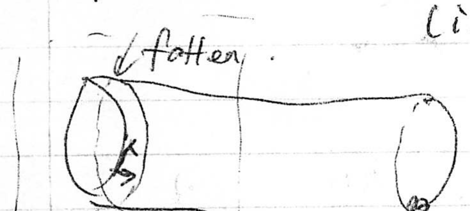
Framed cobordant submanifolds

$N_1$ $N_2$ cobordant in $M$

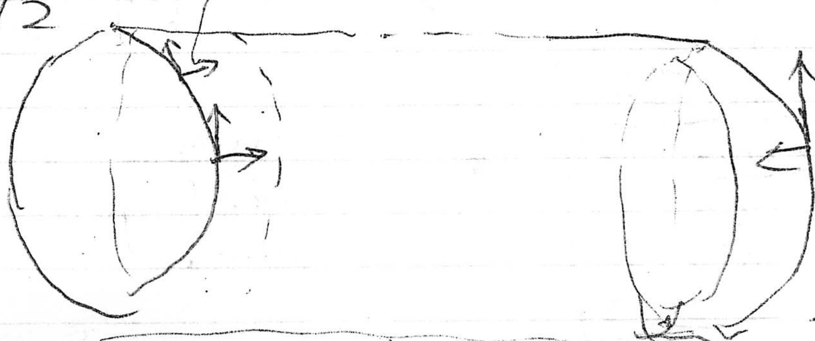
$N_1^{b_1}$ $N_2^{b_2}$ framed in $M$ $b_1, b_2$ choices of basis's of complement

frames are compatible

(i.e. Cobordism $M$ extends $b_1$ to $b_2$)




framing

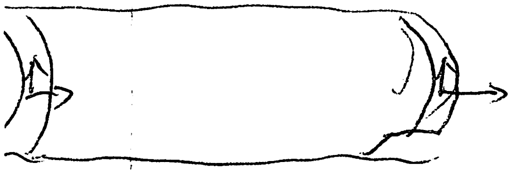


Not a framed Cobordism (Not Compatible with the left)

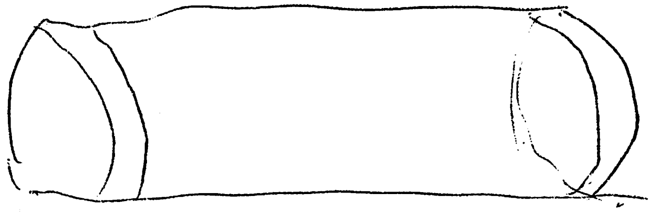
via $M$

$b_1$ $b_2$

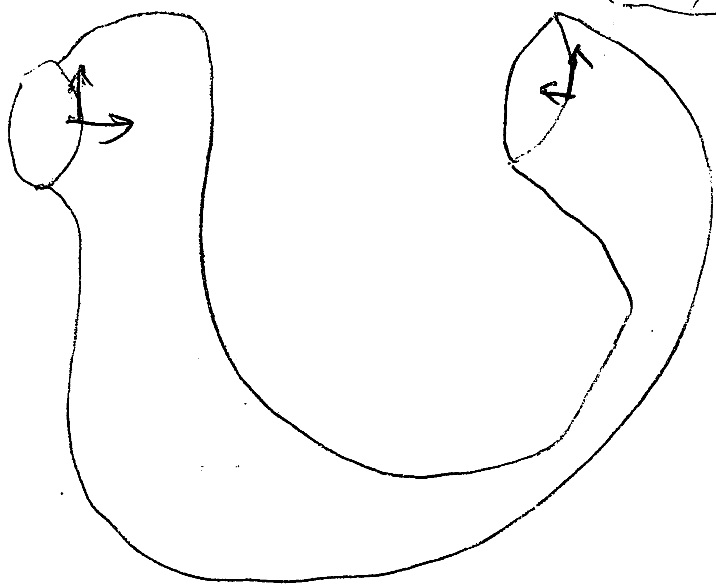
The situation can be fixed by

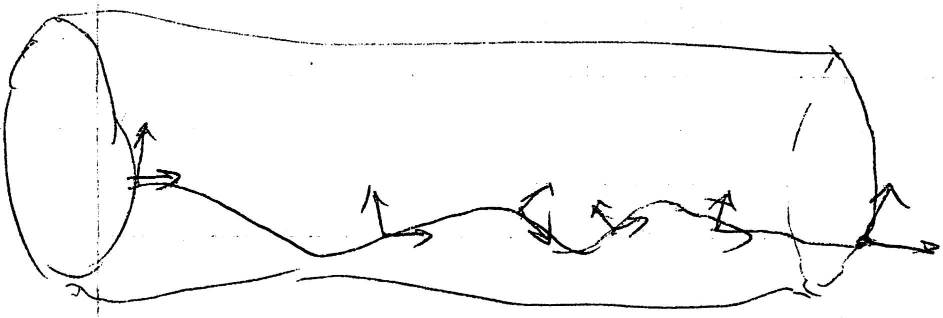


OR

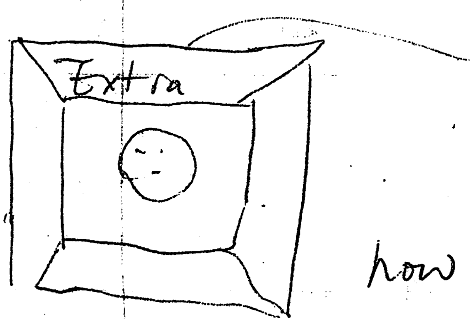


OR





kind of like a homotopy
from $N_1$ to $N_2$
$b_1$ to $b_2$.




"framing is extra structure
to tell you how to look the
picture.

how the sits in a big manifold."

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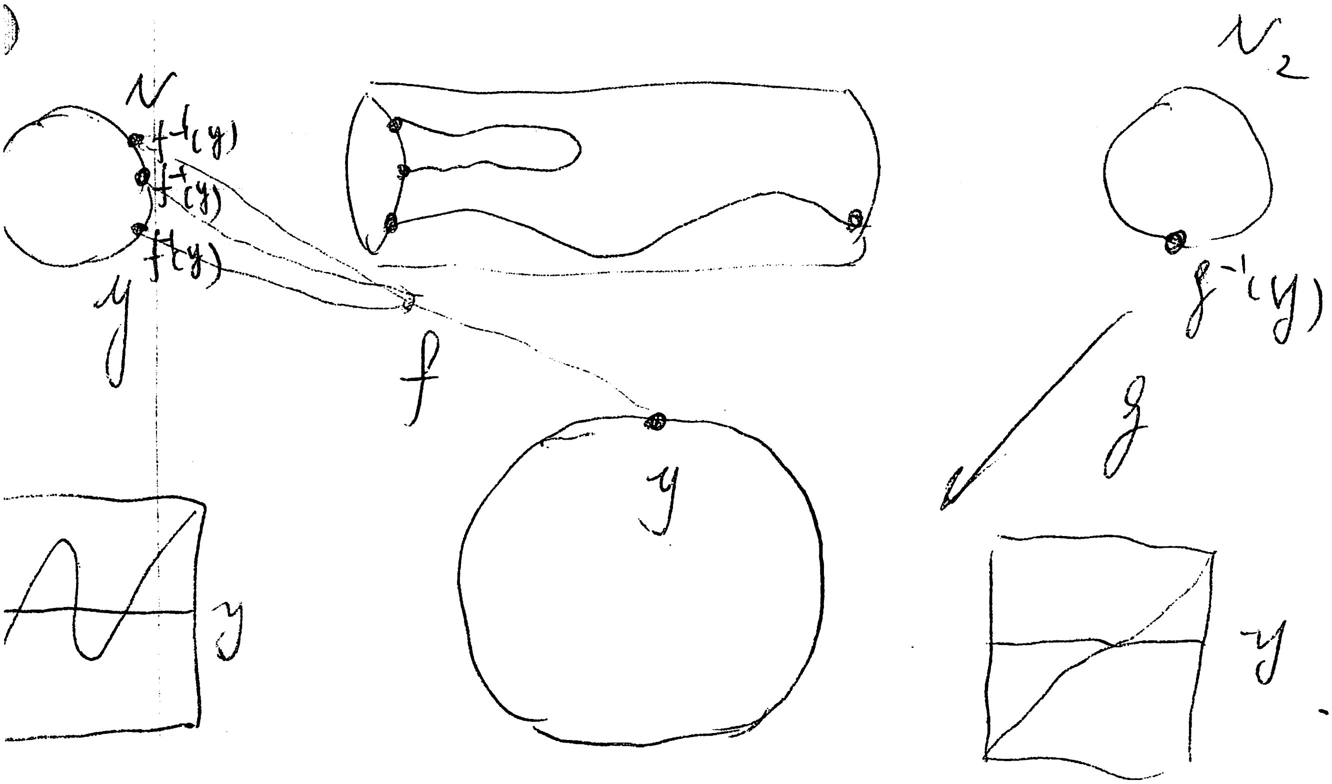
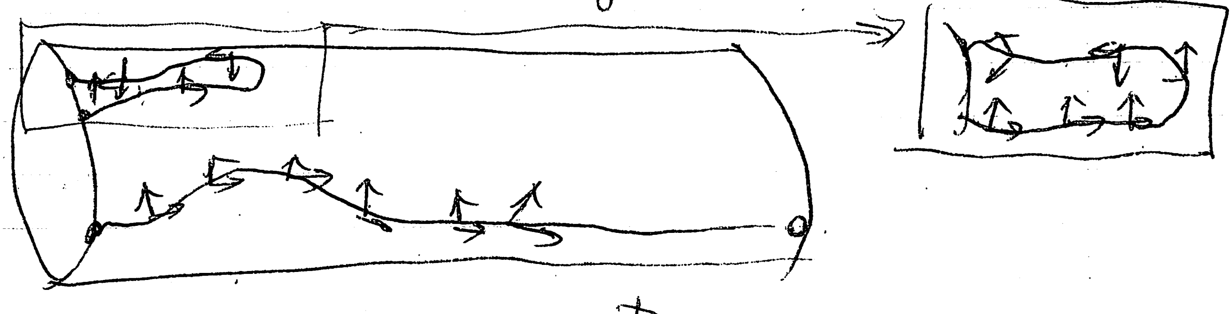



$f \sim g$
via
F

$F(x,0) = f(x)$
$F(x,1) = g(x)$
so $F^{-1}(y)$

But ~~no~~ now we have framing here



Pontraygin.