James Lewandowski
11\21 notes
MAT 364

Euler char

given m compact manifold (w/ ∂)

- Triangulate m

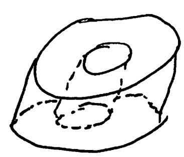
$$\chi(m) = \sum_{j}^{m} (-1)^j H^j(T)$$

j^{th} homology

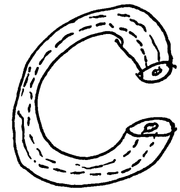
in case $\dim(m) = 2$, (verts) − (edges) + (faces)
joints bones skin

example of 3 manifold m^3

take solid cube, glue opposing faces



right glued to left



= Solid Torus
w/ hole drilled

= jelly donut
w/ no jelly

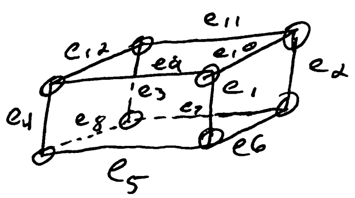
Now glue inside to outside.

Whats $\chi(m^3)$?

have a triangulation

Triangulation is a subdivision into simplicies of dim k

each a k-dim disk (or pt)



(verts) − (edges) + (faces) − (solids)
joints bones skin flesh

− # solids = 1
+ # faces = 3
− # edges = 3
+ # verts = 1

- before gluing
  faces = { R, L, T, B, F, $\overset{B}{\underset{k}{a}}$)
  glue R ⇒ L, T ⇒ B, F ⇒ Back
  to have 3 faces
- before 12 edges

$\bar{e}_1 = e_1 \overset{RL}{\Rightarrow} e_4 \overset{F\,Back}{\Rightarrow} e_2 \overset{RL}{\Rightarrow} e_3$

$\bar{e}_6 = e_6 \Rightarrow e_8 \Rightarrow e_{10} \Rightarrow e_{12}$

$\bar{e}_5 = \left(\begin{array}{l} e_9 \Rightarrow e_{11} \\ e_5 \Rightarrow e_7 \end{array} \right.$

So 3 edges

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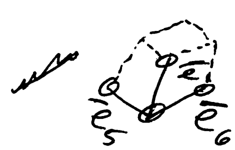
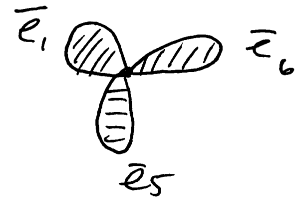


||

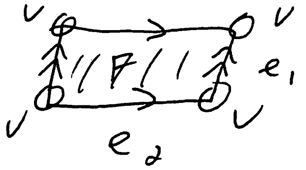
1 vertex (all glued together)

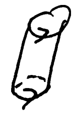


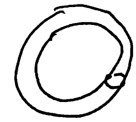
$1 - 3 + 3 - 1 = 0 = \chi(M^3)$

So This is a $T^3 = S^1 \times S^1 \times S^1$

$\cong T^2$



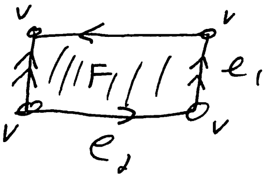


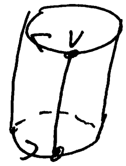


$0 = \chi(T^2) = \chi(K)$

$1 - 2 + 1 = 0$

Klein Bottle

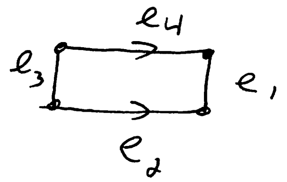




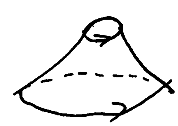
• Same euler char

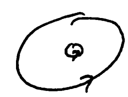
$0 = 1 - 3 + 2$ $\quad F = 1 \quad e = 3$

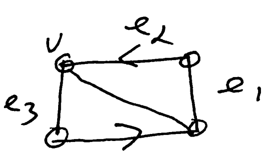
$v = 2$



glue $e_2 \Rightarrow e_4$



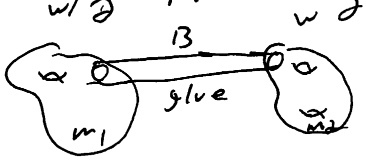




$\chi(m) = -1$

$B_1 = B_2$

m_1 w/ ∂ $\quad \#$ $\quad m_2$ w ∂



$\chi(M_1) + \chi(M_2)$

$- \chi(B_1 = B_2)$

$B_1 \subset \partial m_1$

$B_2 \subset \partial m_2$