11/18

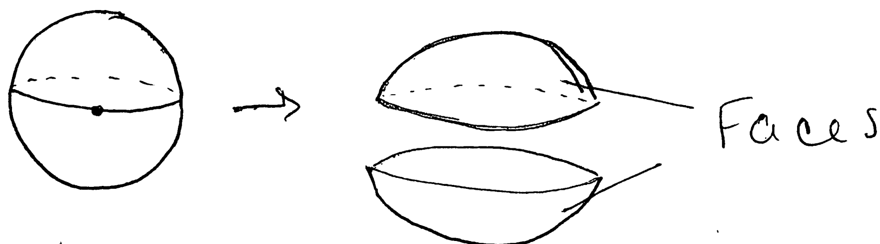
LEEZA

Triangulations of M

(for now, M is a compact 2-manifold)

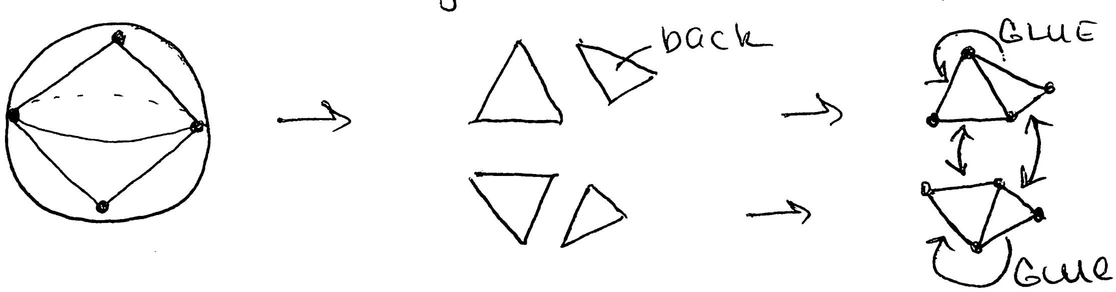
A triangulation of M means we divide up the manifold using points and cuts

- Choose a collection of points (vertices) and arcs (edges) connecting the points so that $M - (\cup v_i \cup \cup e_j) =$ union of disks.

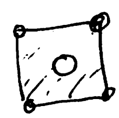


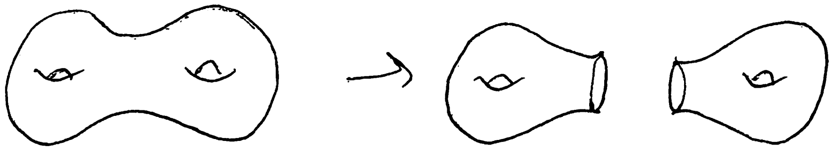
In this minimal triangulation of the sphere we have 1 edge, 1 vertex and 2 faces

Another triangulation of the sphere:

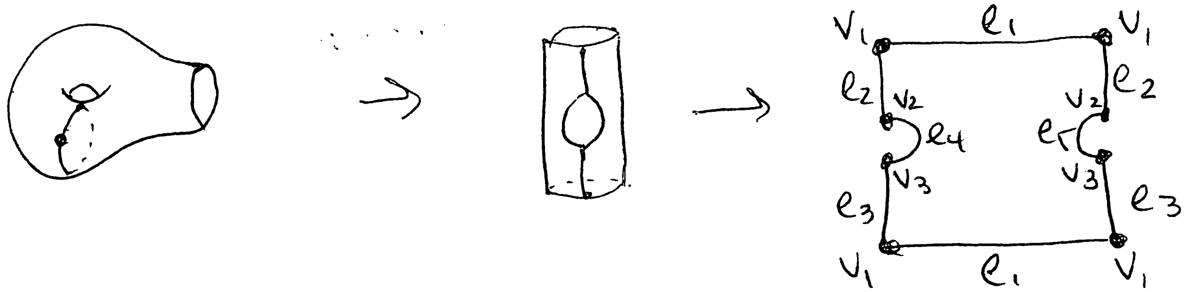


(vertices = 0 skeleton (0 homology)
edges = 1 skeleton (1 homology)
faces = 2 skeleton (2 homology))

Not a triangulation:  (BECAUSE THE FACE HAS A HOLE AND IS NOT A DISK)



Triangulation tells you a way to choose charts on M
we can cut this open further to make disks



Given a triangulation of M, can form a number $\chi_T(M) = \#\text{vertices} - \#\text{edges} + \#\text{faces}$

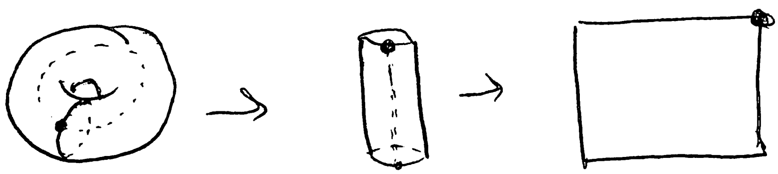
In this example $\chi_T(M) = 3V - 5E + 1F = -1$ (each half)

So $\chi_T(M) = -2$.

Fact: $\chi_T(M)$ doesn't depend on T (any T on the same M gives the same Euler characteristic)

$\chi(S^2) = \chi($  $) = 2 - 1 + 1 = 2$

$\chi(\mathbb{T}^2) = \chi($  $) = 1 - 2 + 1 = 0$



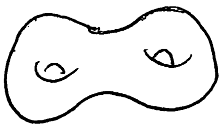
$1V - 2E + 1F = 0$

notice: adding a vertex on the edge turns one edge into 2 and adding an edge on a face turns 1 face into 2

So $\chi(M)$ stays the same.

 $\cong$  (sphere with 3 handles) = genus 3

(torus = genus 1)

 = genus 2

In general, every time we add a genus, χ goes down by 2

$\chi(\text{genus } g) = 2 - 2g$

(cutting a hole takes χ down by 1 and gluing a handle takes χ down by 1)