MAT 364
Class notes

11/9

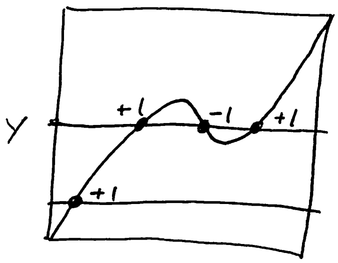
James Lewandowski:

Brover Degree of f:

$f : M \to N$

y is a reg value

$$\deg(f; y) = \sum_{x \in f^{-1}(y)} \operatorname{sign}(\det(df_x))$$

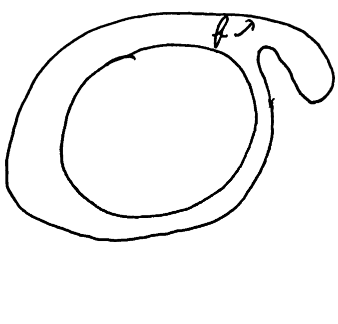


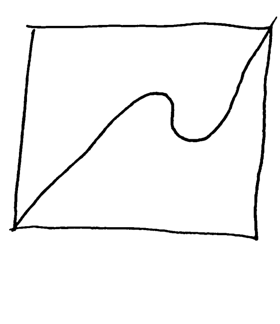
$\deg(f; y) = +1 - 1 + 1 = +1$

[Don't define it for Critical points (for now)]

<u>Thm A</u>: $\deg(f; y)$ Doesn't depend on y (So can write $\deg f$)

<u>Thm B</u>: if $f \sim g$ Smoothly homotopic Then $\deg(f) = \deg(g)$





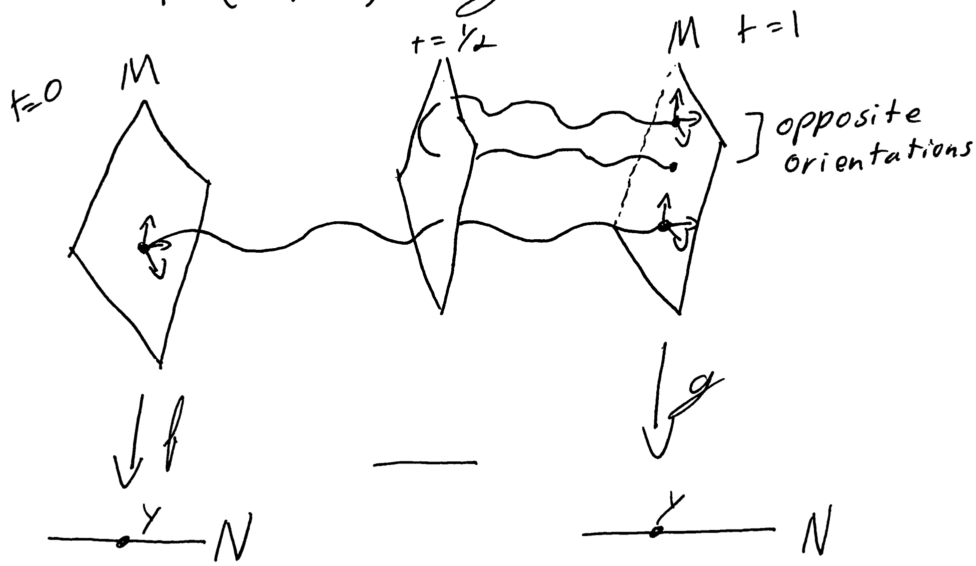
$g, f : M \to N$

Spoze F is homotopy from f to g

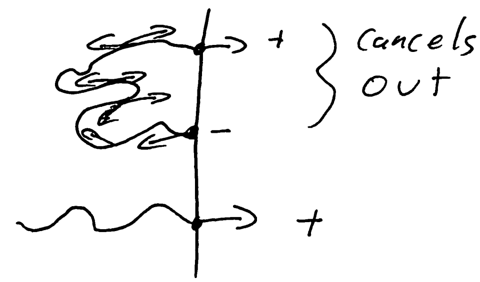
$F : M \times [0,1] \to N$

$F(x,0) = f(x)$

$F(x,1) = g(x)$

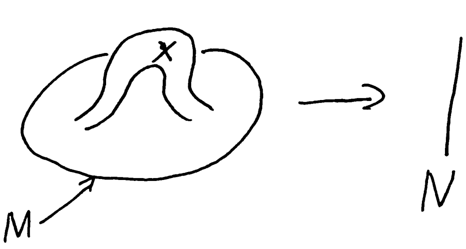


Orientation



X = manifold w/ boundary

$\partial X = M$ = Boundary of X



if $f : M \to N$ (smooth) extends to $F : X \to N$ (smooth)

Then $\deg(f; y) = 0$ at every reg value $y \in N$.

p.2 | Aside

remember: Compare to Lemma in proof of Brover F. P.

(i.e $\deg f \neq 0$) if $f: S^n \to S^n$ has no fixed pt Then f cannot extend to Smooth $f: \overline{D}^{n+1} \to \overline{D}^{n+1}$

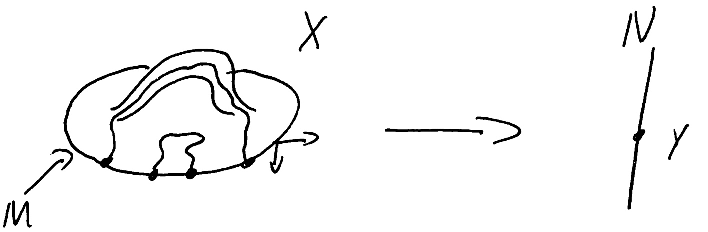
aside end.

Spoze $f: M \to N$ extends to $F: X \to N$

take y regular for f

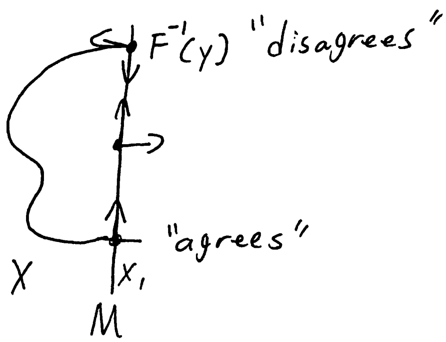
- if y is also regular for F

$F^{-1}(y)$ is a 1-manifold w/ Boundary, + Boundary lies in M.



Orientation for X

So that points out along M.



So $\deg(F(y)) = 0$

This gives orientation for $F^{-1}(y)$

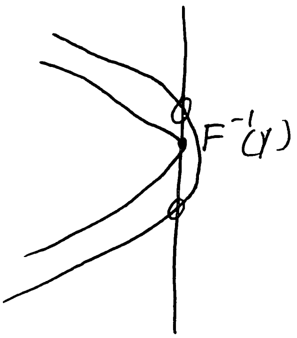
pick some $x \in M \cap F^{-1}(y)$

So

$\text{Sign}(\det(dF_{x_1})) \neq \text{Sign}(\det(dF_{x_2}))$

i.e: "Cancels at each end"

if y not reg for F (But is for f)



move y a little along N,

X "Splits"

points regular for F

(by Sard's Thm)

Use same argument

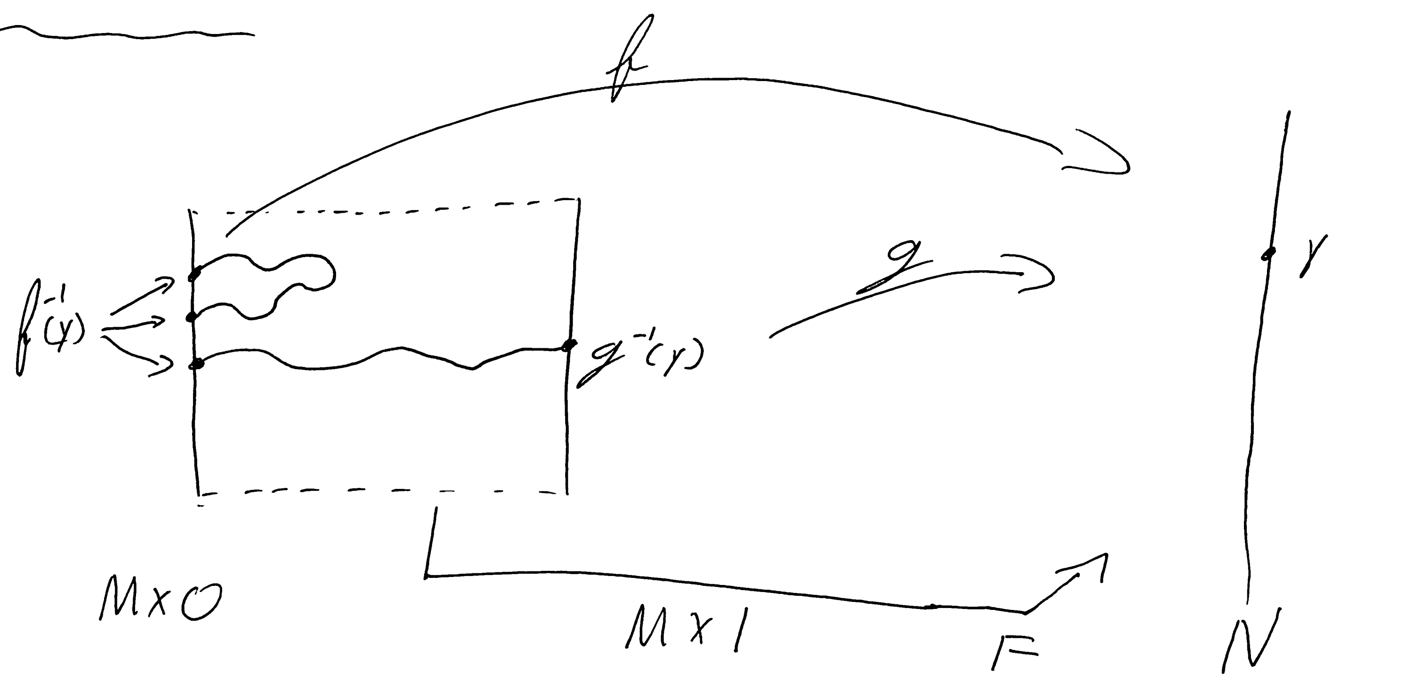
To see that if

$(f, g: M \to N)$ $\quad f \sim g$, $\deg(f) = \deg(g)$

by applying Lemma

if $f \sim g$, $\exists\ F: M \times [0,1] \to N$

w/ $F(x,0) = f(x)$, $F(x,1) = g(x)$



<u>Cor</u>: Can't comb hair on a billiard ball.

(next class)