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Notes 11/7

Orientation Continued

Here we will be looking to find a correlation between orientation and vector spaces.

Let V be a vector space of dim n.

Let $\{v_1, v_2, v_3, \ldots, v_n\} = \beta$ be a basis.

Another basis $\{w_1, w_2, w_3, \ldots, w_n\}$ has the same orientation if we can write:

$$w_i = \sum_{i=1}^{n} a_i \, v_i$$

$$A = \begin{matrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & & \vdots \\ a_{m,1} & \cdots & a_{m,n} \end{matrix}$$

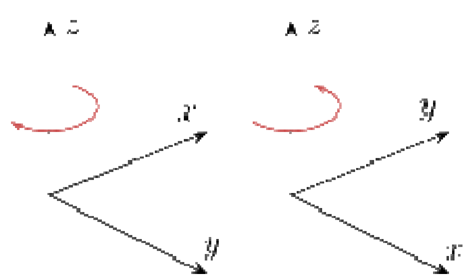
If the det $A > 0$, the then two basis have the same orientation.

In general, given a linear map

$A : V \rightarrow W$, V and W are vector spaces,

We say A is orientation preserving if the det $A > 0$, and we say A is orientation reversing if the det $A < 0$.

We can think of this as being analogous to multiplying a number by -1 an odd number of times changes its sign, but multiplying by -1 an even number of times does not change the sign.

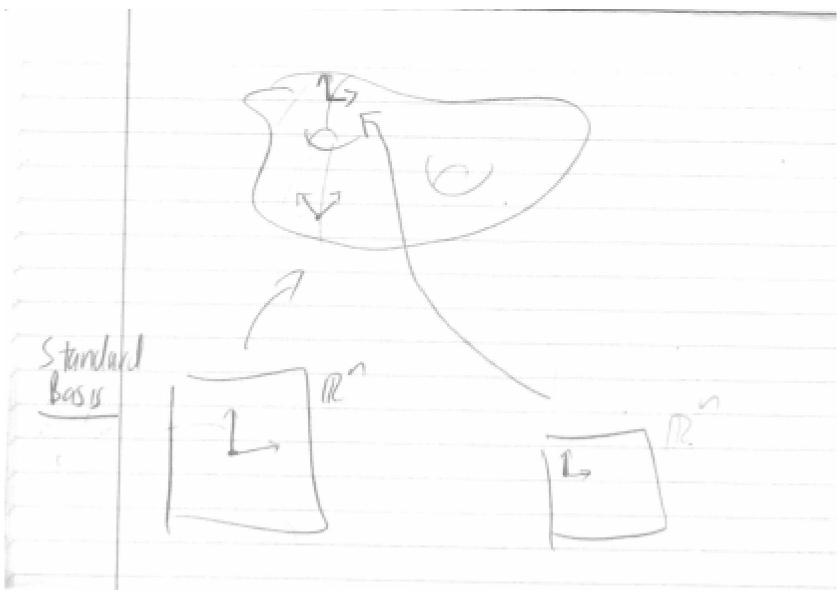


In the diagram above we have the *xyz* axes with different orientations. If we were to flip the orientation of two of the axes, we would be back to the original orientation.

Since this idea is based off of a linear map A, how do we relate this to our topology?

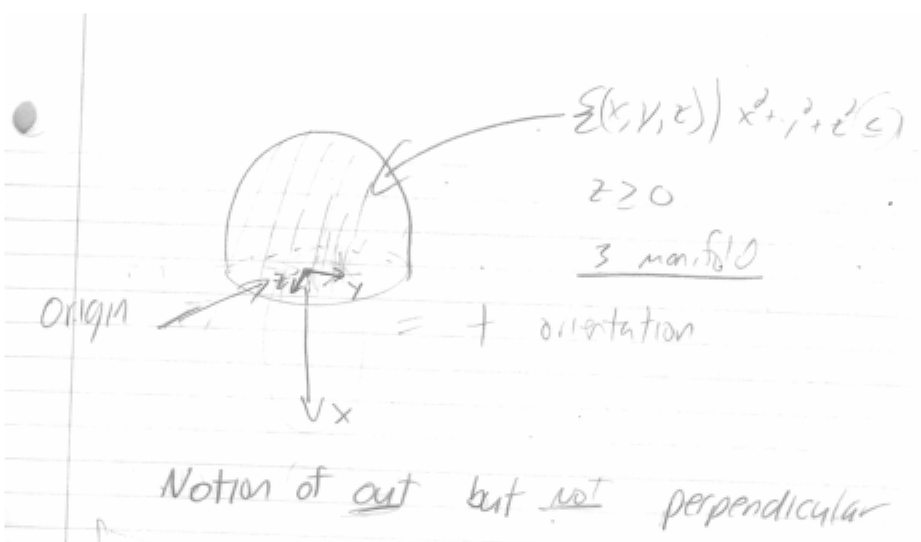
If $f : M \rightarrow N$ is smooth at some point x, we say that f is orientation preserving at x if:

df_x is an orientation preserving map, i.e. if $df_x > 0$



If M is a manifold with boundary, we can orient the boundary as follows:

A positive orientation has one of the vectors, the "first" vector pointing out, and the rest of the vectors will be tangent to the boundary.



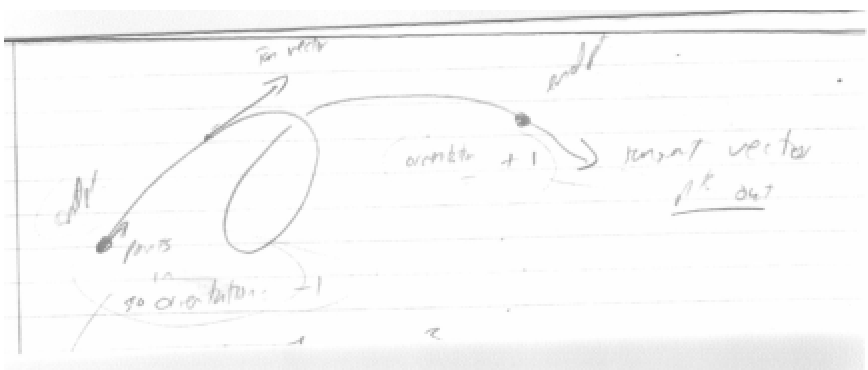
What is in the brackets above came out light. It is:

$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$

Example

Let γ be a one manifold.

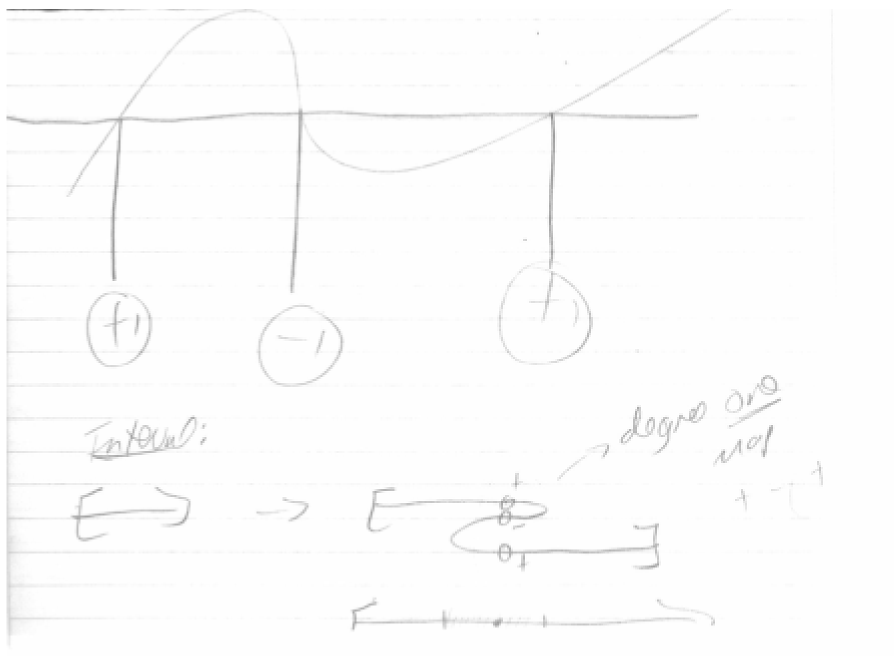
γ: [0,1]→R^3



We can see that if we start at one end of the curve, place the first arrow pointing "in" (going in the direction of the curve), and call that the positive orientation, the orientation remains constant until the second endpoint, where the orientation is reversed since it is now pointing "out".

Example

f : [0,1] → [0,1] as shown below, is an orientation preserving degree one map



Brouer Degree of f at y

Given a smooth map:

$f : M \rightarrow N$

and y a regular value for f, the Brouer degree of f at y is:

$$\deg(f\,;y) = \sum_{x\in f^{-1}} sign(df_x)$$

Claim: deg (f ; y) does not depend on y.