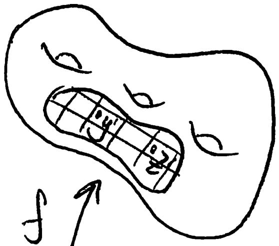
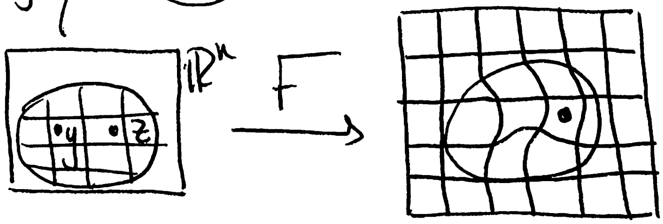
11/04/11

LEERA

Within a coordinate patch all points are isotopic



Can find a deformation that moves y to z and doesn't touch anything around it (can do this smoothly)



Consider the problem of an isotopy F_t of $\mathbb{R}^n$ sending some y to some chosen z with $y, z \in D_1$ and F_t smooth. $F_t(x) = x$ for $\|x\| \geq 1$

write a smooth ODE with initial value $(t=0)$

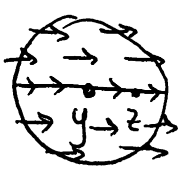
solution: $\varphi_t(0) = y$ and $\varphi_t(1) = z$

The way to do it:

without loss of generality assume $y = 0$

can rotate disk so that $z = (r, 0, 0, 0 \ldots)$

$(0 < r < 1)$

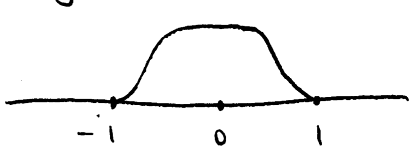


Can take DE which has

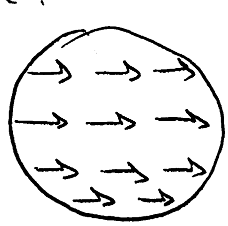
$\dot{x}_1 = \text{constant} > 0$

$\dot{x}_j = 0, \quad j \neq 1$

The only problem with this DE is that it affects the outside of the disk so need to adjust DE by multiplying each vector by a smooth function whose graph looks like:



so now we have:



same solution inside the disk but nothing changes outside so we get a family of functions depending on what t is

for each t, $F_t(x) = \alpha(t, x)$

smooth so it's a diffeomorphism.

$F_1(y) = z$ $\quad$ $F_0(x) = x$

varies smoothly in t as well so it gives us an isotopy from y to z

<u>Lemma</u>: if N is a smooth, connected manifold, any point $y \in N$ is isotopic to any other point $z \in N$.

<u>Theorem</u>: if y, z are regular values of $f: M \to N$ where N is connected, ~~___~~ and $\dim M = n$ $\dim N = n$

then $\# f^{-1}(y) \equiv \# f^{-1}(z) \mod 2$.

pf: we already have pf for case where

$\# f^{-1}(y) \equiv \# g^{-1}(y) \mod 2$ if $f \sim g$.

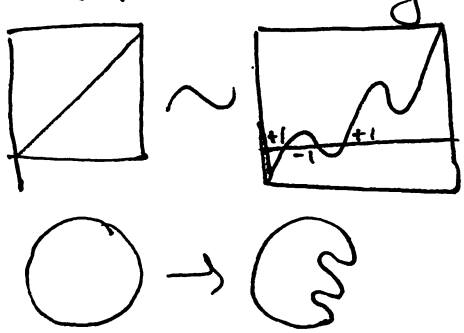
and now we can just take the identity and isotope y to z.

So this means "odd degree function"

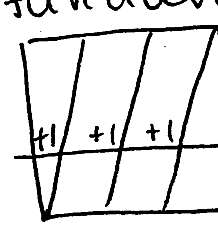
and "even degree function"

is a topological property

Want to say $f: S^1 \to S^1$ is degree 1



and is fundamentally different from

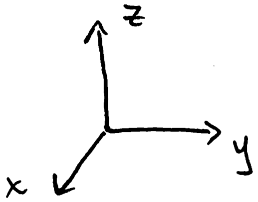


which has degree 3

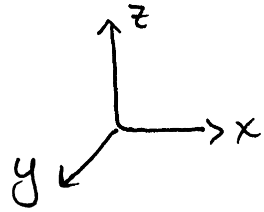
(wraps around 3 times)

Idea: want to make sense of the notion of degree but want to have signs.

to make this work, we have to introduce orientation on M.



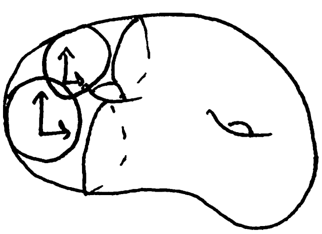
Right-handed coord. system for $\mathbb{R}^3$



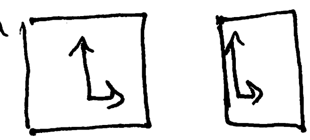
Left handed coord system for $\mathbb{R}^3$

(can choose different orientation)

Can choose orientation on manifold



pick some orientation at base point and "push it around" using overlap of the charts

$\mathbb{R}^n$ 

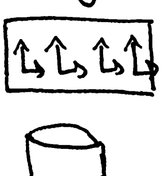
go from one chart to the next making sure they agree.

2 possibilities: ① everything agrees

or

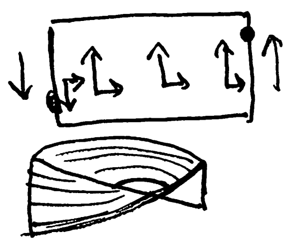
② it doesn't.

cylinder:



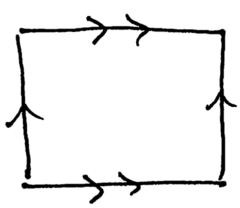
(the charts agree)

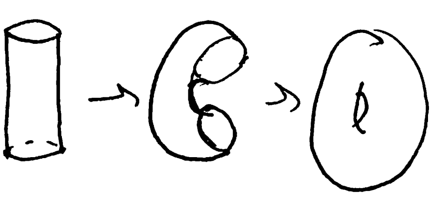
mobius strip:



charts don't agree (not orientable)

torus:

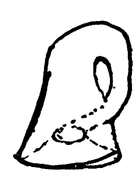




orientable

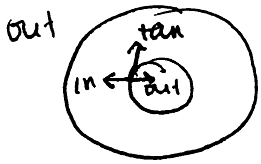
klein bottle:

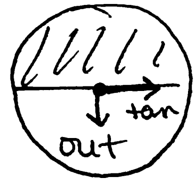
mobius strip inside klein bottle



not orientable

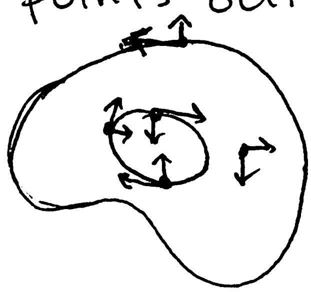
if orientable manifold has boundary
can prefer one orientation
since there is an inside and an outside
there's a vector we can tell apart
(vectors that point out and vectors
that point tangentially)





$\mathbb{H}^n$

can change our vectors so that all of them
point tangentially except the one that
points out



the two curves need to
have different orientations.