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We are trying to make sense of the degree of a function in a topological way!

FIRST STEP:

Both M and N are connected and compact without boundary.

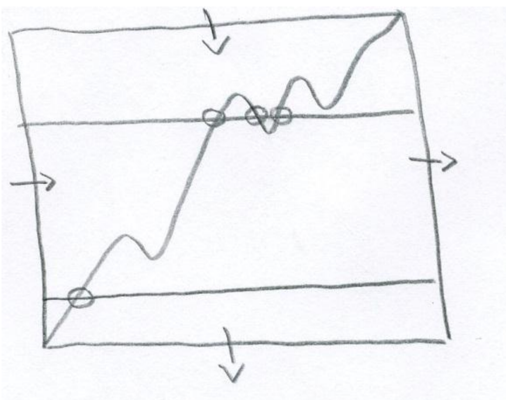
$f: M \to N$ at regular points $y \in N$ and $z \in N$

We want $\# f^{-1}(y) \equiv \# f^{-1}(z) \bmod 2$.

In English this means they are either both even or both odd.

Proto Type:

$f: S^1 \to S^1$



$3 \equiv 1 \bmod 2$

The function f is homotopic to the identity (we can straighten the line out).

This is how far we got (before the Midterm):

$f, g: M \to N$ with $f \sim g$ (f smoothly homotopic to g)

and M compact without boundary $y \in N$ and dimM=dimN

then $\# f^{-1}(y) \equiv \# g^{-1}(y) \bmod 2$

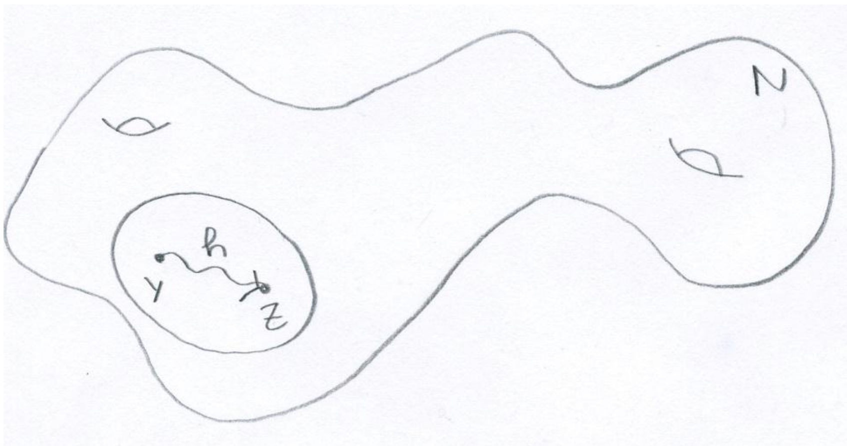
<u>Homogeneity Lemma:</u>

If N is a smooth and connected manifold with $y, z \in IntN$

then there is a diffeomorphism $h: N \to N$ with $h(y) = z$

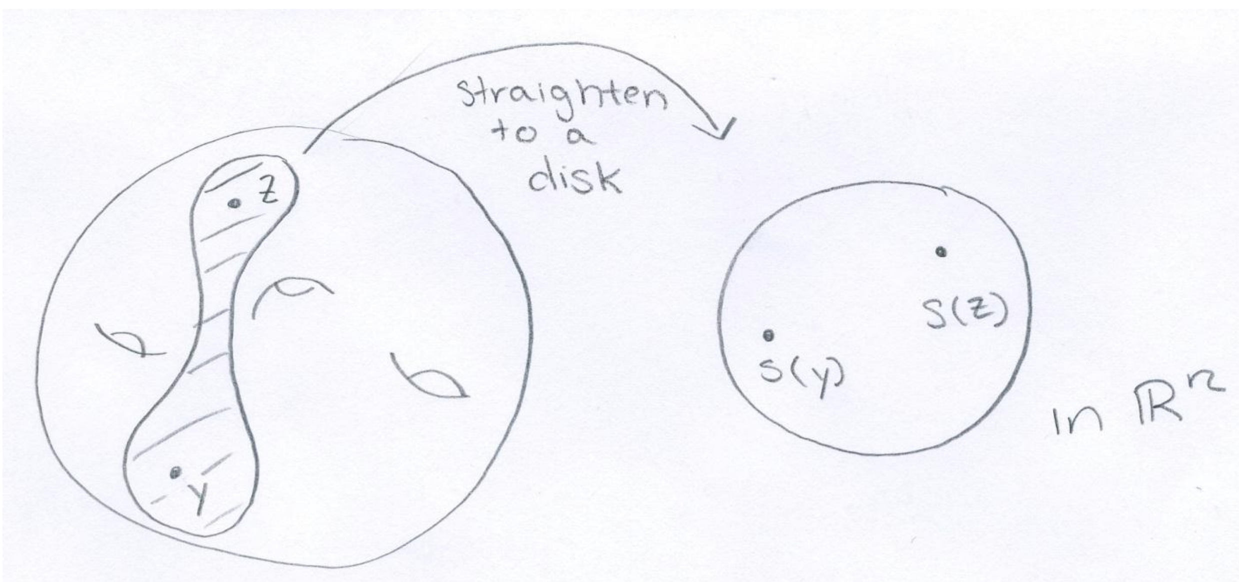
(h is smoothly isotopic to the identity).

i.e.

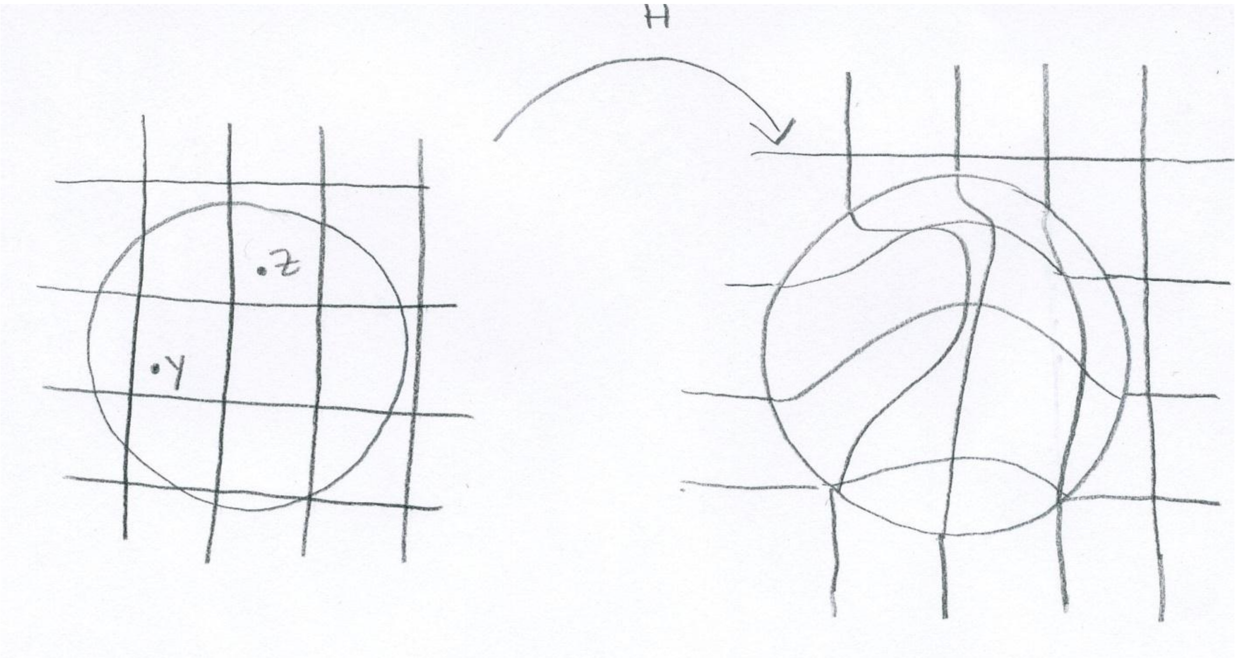


What does this mean? Take y and drag it over to z!

Since N is connected we can find an open disk $\cong$ to the n-disk, which contains y and z.



Now we need to find $H: R^n \to R^n$ so that outside of the disk $H(x) = x$ and inside the disk $H(y) = z$ and is homotopic to the identity.



We need to prove that we can do this smoothly.

Let $\varphi: R^n \to R^n$

$\varphi(x) > 0$ for $\| x \| < 1$

$\varphi(x) = 0$ for $\| x \| \geq 1$

$\varphi(x) = \lambda(t)(1 - \| x \|^2)$

$$\lambda(t) = \begin{cases} 0 & if \ t \leq 0 \\ e^{-\frac{1}{t}} & if \ t > 0 \end{cases}$$

To find H we can define a differential equation (DE), so that for each $c \in S^{n-1}$

$$\frac{dx_i}{dt} = c\varphi(x_1, ..., x_n)$$

This smooth DE will describe a flowing and have a unique solution.