Zelong Li's NOTE

MAT 364 "HOMEWORK" BY FRIDAY.

- SUGGEST A PROBLEM FOR THE MIDTERM!

STILL PROVE "SARD THM"

U OPEN $\subseteq \mathbb{R}^n$, $f: U \to \mathbb{R}^p$ SMOOTH, $C = \{$CRIT PTS OF $f\}$

THEN $f(C)$ HAS MEAS 0 IN $\mathbb{R}^n$.

$C \supseteq C_1 \supseteq C_2 \supseteq C_3 \cdots$ "NESTED"

$C_1 = \{x \in \mathbb{R}^n \mid df_x = 0\}$

$C_n = \{x \in \mathbb{R}^n \mid$ ALL PARTIALS TO ORDER n ARE ZERO AT $x\}$.

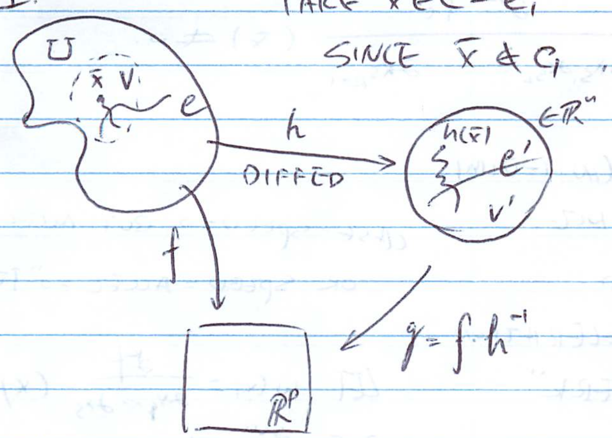
RECALL: STEP 1. $f(C - C_1)$ HAS MEASURE 0

STEP 2. $f(C_n - C_{n+1})$ HAS MEASURE 0

STEP 3. FOR k LARGE, $f(C_k)$ HAS MEASURE 0.

STEP 1. TAKE $\bar{x} \in C - C_1$

SINCE $\bar{x} \notin C_1$, $df_x \neq 0$ AT $\bar{x}$ (SAY $\frac{\partial f_1}{\partial x_1} \neq 0$)



$h(x_1, \ldots x_n) = (f(x_1), x_2 \ldots \cancel{\ } x_n)$

HERE $h: V \to V'$

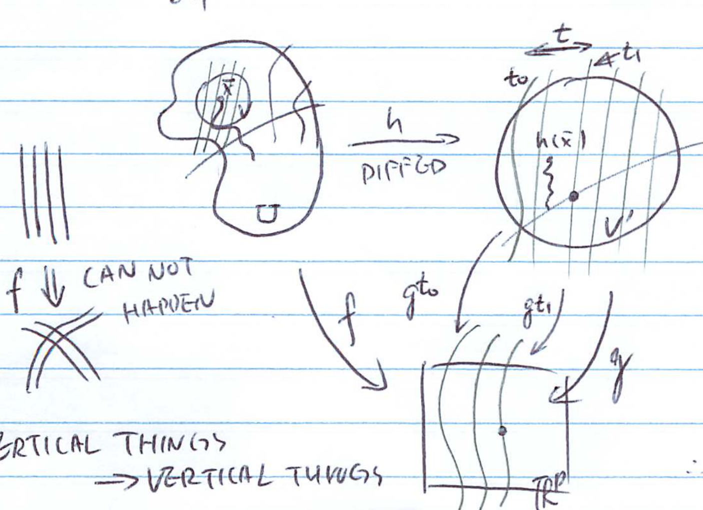
$C' =$ CRIT PTS of $g = h(C \cap V)$

AND $g(C') = f(C \cap V)$

CONSIDER IN 1-DIM

FOR EACH POINTS $(t, x_2, \cdots x_n) \in \mathbb{R}^n$ (FIX t) $\Rightarrow g(t, x_2 \cdots x_n) \in t \times \mathbb{R}^{n-1}$

SINCE $\frac{\partial f}{\partial x_1} \neq 0$.



SO g CARRIES HYPERPLANES IN $\mathbb{R} \times \mathbb{R}^{n-1}$ TO HYPERPLANES IN $\mathbb{R} \times \mathbb{R}^{p-1}$

$\therefore g^t: \{t\} \times \mathbb{R}^{n-1} \to \{t\} \times \mathbb{R}^{p-1}$

IF y IS A CRIT PT FOR g, IT IS ALSO A CRIT PT FOR g^t

$\therefore \frac{\partial g^t}{\partial x} = \begin{pmatrix} 1 & 0 \\ * & \boxed{\frac{\partial g_i^t}{\partial y_j}} \end{pmatrix}$

VERTICAL THINGS $\to$ VERTICAL THINGS

RECALL FUBBINI THM: IF YOU HAVE MEAS ~~ON TH~~ 0 IN ~~THE~~ EVERY SLICE, THEN YOU HAVE MEAS 0 IN EVERYTHING

HAVE MEAS $((e - e_1) \cap V) = 0$; CAN COVER U BY COUNTABLE # OF SUCH V_i

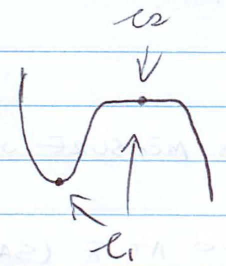
SO MEASURE OF $f(e - e_1) = f\left(\bigcup_{i=1}^{\infty} V_i \cap (e - e_1)\right)$

— COUNTABLE UNION OF MEAS 0 IS MEAS 0.

STEP 2. SHOW $f(C_k - C_{k+1})$ HAS MEASURE 0.

ALL PARTIAL TO ORDER k

TO ORDER $k+1 = 0$.



FOR EACH $\bar{x}$ IN $C_k - C_{k+1}$, WE HAVE SOME

$$\frac{\partial^{k+1} f_r}{\partial x_{s_1} \partial x_{s_2} \cdots \partial x_{s_{k+1}}}(\bar{x}) \neq 0.$$

LARRY'S EXAMPLE: (IN 1-DIM)

$f(x)$ = DISTANCE

$f'(x)$ = SPEED

$f''(x)$ = ACCELERATION

$f'''(x)$ = "JERK"

CASE: SPEED = 0 BUT ACCEL $\neq 0$

OR SPEED = ACCEL = 0 "JERK" $\neq 0$

LET $w(x) = \frac{\partial^k f}{\partial x_q \cdots \partial x_s}(x)$ SO THAT $w(\bar{x}) = 0$

BUT $\frac{\partial w}{\partial x_1}(\bar{x}) \neq 0$ WHERE w MAY BE SPEED OR ACCELERATION