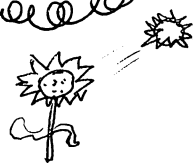
Mat 364
10/12/11

Notes



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Sard's Theorem:

$f: U \to \mathbb{R}^p$, smooth U open in $\mathbb{R}^p$,

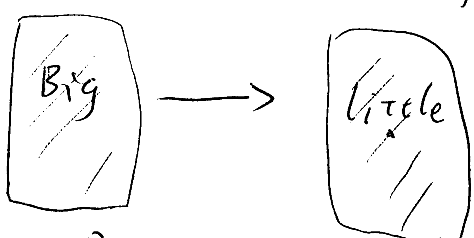
$e = \{ x \in U \mid \text{rank } df_x < p \}$.

Then $f(e) \subset \mathbb{R}^p$ has measure (area) 0.

Not "too many" critical values

might want measure $(e) = 0$,

but no, since $f(U) =$ constant .



All of U is a critical point, but $f(U) =$ point.

$A \subseteq \mathbb{R}^p$,

measure $(A) \geq K$

$\Uparrow$

p-dim volume $(A) = K$.

meas $(pT) = 0$.

(countable union of measure 0 sets is measure 0).

$A \subseteq \mathbb{R}^p$ has measure 0

$\Leftrightarrow A \cap (\mathbb{R}^{p-1} \times pt)$ has measure 0.

"Middle Thirds cantor set"

$C_0 = [0, 1]$

$C_1 = [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$

$C_2 = [0, \frac{1}{9}] \cup [\frac{2}{9}, \frac{1}{3}] \cup [\frac{2}{3}, \frac{7}{9}] \cup [\frac{8}{9}, 1]$.

To go from C_n to C_{n+1}, remove middle third of each interval in C_n.

$C = \bigcap_{n=0}^{\infty} C_n$.

$C \neq \emptyset$, C contains endpoints of all intervals of C_i for each i.

$C = \{ x \in [0, 1] \mid$ base 3 expansion of x can be written without using 1 $\}$.

$$x = \sum_{i=1}^{\infty} \frac{a_i}{3}, \quad a_i \in \{0, 2\}.$$