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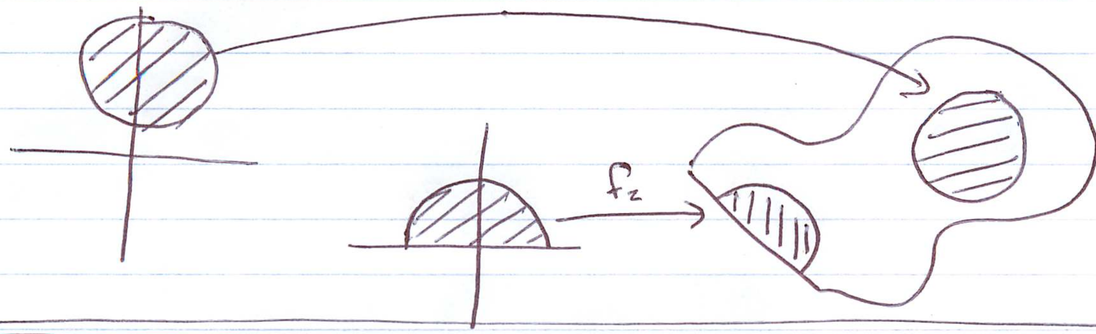
$\mathbb{H}^n$ = Upper half plane of dim n

$= \{ x_1, \ldots, x_n \mid x_n \geq 0 \}$

ex) 

Def: M is a manafold w/ both boundary if $\forall x \in M$, M has NBHD Diffeomorphism to $\mathbb{H}^n$



$A = \{ x^2 + y^2 \leq 1 \}$

(1,0) is in interior (A) relative to A

but

Not in Interior (A) in relation to $\mathbb{R}^2$

b/c then we can step outside of disk

Another example:

$cl(\mathbb{R}^2) = \mathbb{R}^2$ Rel to $\mathbb{R}^2$

Can think of $\mathbb{R}^2$ "inside" S^2

$S^2 - \{\text{north Pole}\} \cong \mathbb{R}^2$

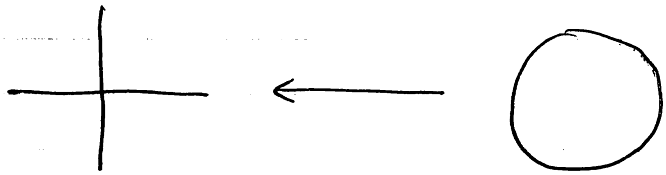
$\mathbb{R}^2 \cong$ Open disk

$\frac{z}{1-z} \longleftarrow z$

$Cl(\text{open disk}) = $ Closed disk

$\frac{re^{i\theta}}{1-r} \longleftarrow re^{i\theta}$

$\{x^2+y^2 \leq 1\}$



"Adding a circle at infinity"

$\left.\begin{array}{l} cl(\bar{D}) = \bar{D} \\ cl(S^2) = S^2 \end{array}\right\}$ "why is this true relative to Anything?"

"b/c there is No possibility of Additional boundary"

$\underline{\underline{\text{Def}}}$ if $A \subseteq \mathbb{R}^n$, A is Compact if A is closed and bounded

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Can we capture this in terms of a sequents of points?

ie limit Points

$(0,1)$ is Not compact b/c its Not closed

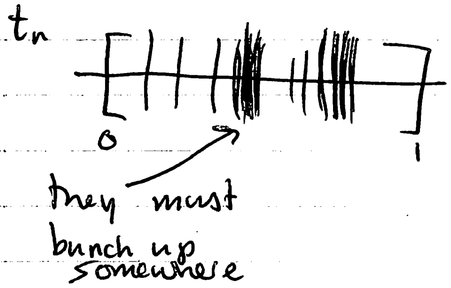
but $cl((0,1)) = [0,1]$

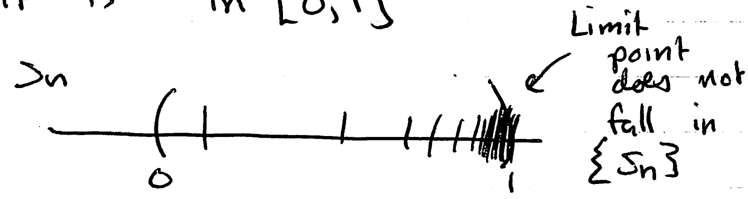
Consider A sequence $\{s_n\}_0^{\infty}$ $s_n \in (0,1)$
then we can have $s_n \to 1$

Compared to $\{t_n\}_0^{\infty}$ $t_n \in [0,1]$

$\Rightarrow$ given any $\{t_n\}_{n=1}^{\infty}$ there is a limit point in $\{t_n\}$

and the limit point is in $[0,1]$





$\mathbb{R}^1$ $\{n\}_{n=1}^{\infty}$ Does not have subsequence in $\mathbb{R}^{\circ}$

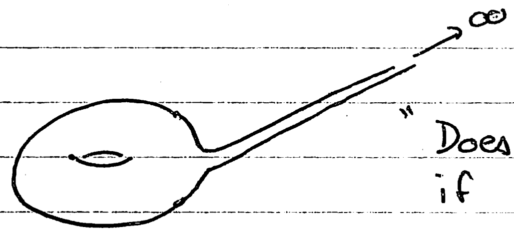
$\mathbb{R} \cup \{\infty\} \cong \bigcirc$ $\Rightarrow$ going far to left = going far to right
can compactify $\mathbb{R}$ by adding a point at $\{\infty\}$

$\Rightarrow$ If I have any sequence of Real #s
- either it has a limit point in $\mathbb{R}$
- or Not $\Rightarrow \infty$ is limit point

is $\mathbb{C} - \{0\}$ compact? No b/c seq $\to 0$ ie $\{\frac{1}{n}\}_{n=1}^{\infty}$
No limit point

and Not closed.

is $\mathbb{C}$ compact? No b/c $\{n\}_{n=1}^{\infty} \Rightarrow$ limit point needed at infinity



"Does it have an end?"
if No then its not compact

<u>Good stuff about compact:</u>

- every seq has a convergent subsequence
- if f is a homomorphism and A is compact $f(A)$ is compact too