Lecture 4 9/9/11

$\boxed{\text{No Class Fri 9/16}}$

Remember
Recall open, closed ...

A sequence $\{x_i\}_{i=1}^{\infty}$ is a ordered list of points

$x_1, x_2, \ldots$

$1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n} \ldots$

$\neq \{x \in \mathbb{R} \mid \exists n \in \mathbb{N}\ x = \frac{1}{n}\}$

DEF A sequence $\{x_i\}_1^{\infty}$ ~~has a limit~~ if Every

x is a limit point of

Neighborhood of x
Contains infinitely many x_i

Example.

$\{1, \frac{1}{2}, 1, \frac{1}{3}, \frac{1}{4}, 1, \frac{1}{5} \ldots\}$

Has 0 as a limit pt.
Has 1 as a limit pt.
$\frac{1}{2}$ is not a limit pt of the sequence.

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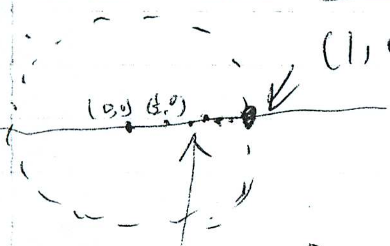
limit of $\sin(n)$ is $[-1, +1]$
Morally easy, techanicaly difficult.

limit of $\sin(\frac{\pi}{2}n)$: $\{0, 1, -1\}$

Proposition. If x is a limit point $A \subseteq \mathbb{R}^n$
Then there is a sequence
$\{x_i\}_{i=1}^{\infty}$ with $x_i \in A$
s.t. x is a limit point of $\{x_i\}$

Example



$\mathbb{D}$ = unit Disk in $\mathbb{R}^2$

$x = (1,0) \notin \mathbb{D}$
x is a limit point of $\mathbb{D}$
Based on the proposition, can find
$x_i \to (1,0)$ w/ $|x_i|^2 < 1$

YEP: Example $\{(1-\frac{1}{n}), 0)\}_{n=1}^{\infty}$ works

Proof: CASE I, $x \in A$.
use seq $\{x, x, x \cdots\}$
CASE II, $x \notin A$
It must be true that
$x \in Fr(A)$
So, $\forall D_r(x) \cap A \neq \emptyset$ has some $x_r \in A$
So construct the sequence
$\{x_1, x_{1/2}, x_{1/3} \cdots\}$

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The reverse is true, too:

If $\{x_i\}$ has x as a limit point, & $x_i \in A \Rightarrow x$ is a limit point of A.

$\therefore$ x is a limit point $A \subseteq \mathbb{R}^n$

$\Longleftrightarrow$

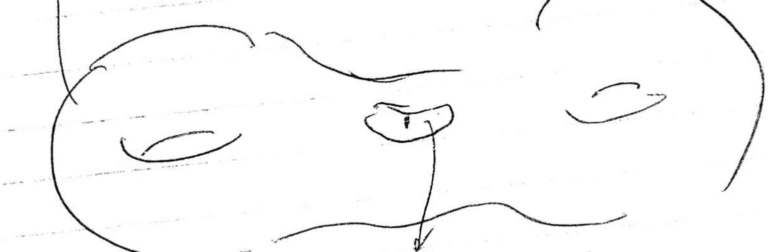
$\exists \{x_i\}_1^\infty$ with $x_i \in A$ s.A. x is a limit point of $\{x_i\}$

Recall

A set is <u>Countably infinite</u>,
If $\exists$ a function f s.t.
$f(\mathbb{N}) \to A$
that is bijective.

If you cannot find such f. it is uncountable

Motivation



<u>DEF</u> $x \in A \subseteq \mathbb{R}^n$.
<u>A relative neighborhood of x in A</u>
is $D_r(x) \cap A$
This is open Rel A

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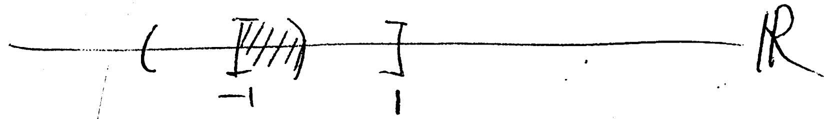
Example. $[-1, 0)$ is open Rel $[-1, 1]$.



FACT: A is open REL A.

$\forall A \subseteq \mathbb{R}^n$ A is closed REL A.

<u>Def</u>: $f: X^{\subseteq \mathbb{R}^n} \longrightarrow Y^{\subseteq \mathbb{R}^n}$ is continous.

If $\forall$ open set $U \subseteq Y$

$\underbrace{f^{-1}(U)}_{\text{set in } X}$ is open.

Recall if $f: A \to B$ and $U \subseteq B$.

The set $f^{-1}(U) = \{ x \in A \mid \exists y \in B$ with $f(x) = y$