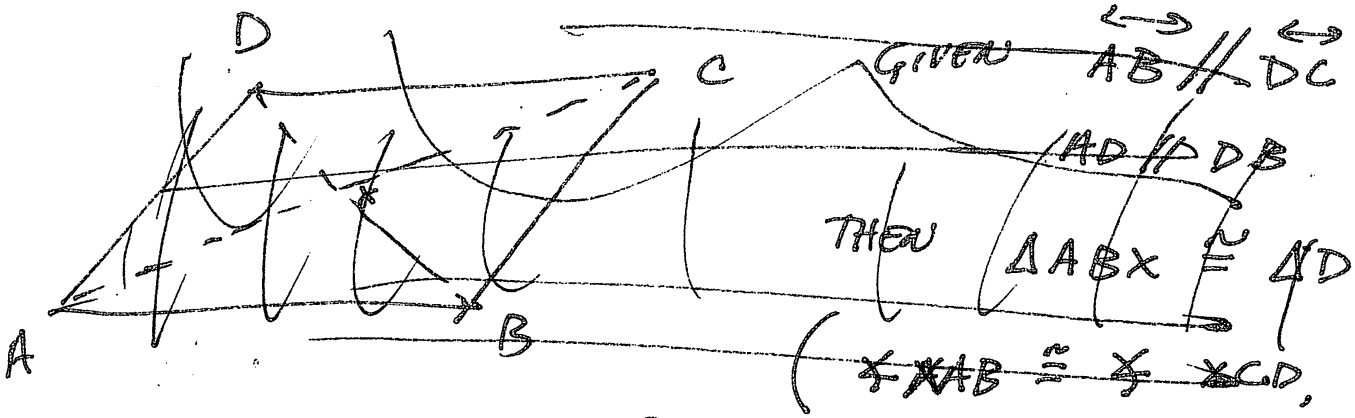
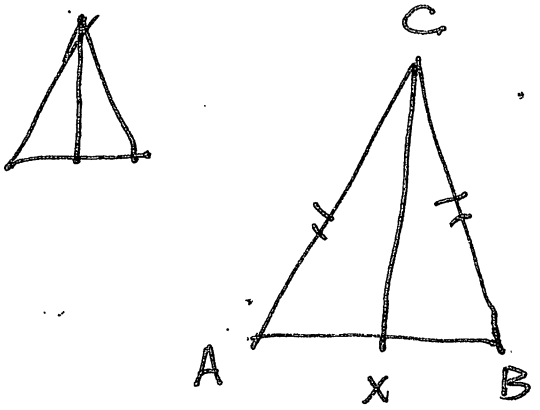
• WHAT IS GEOMETRY ? WHAT IS TOPOLOGY?

IN GEOMETRY, NOTION OF CONGRUENT.





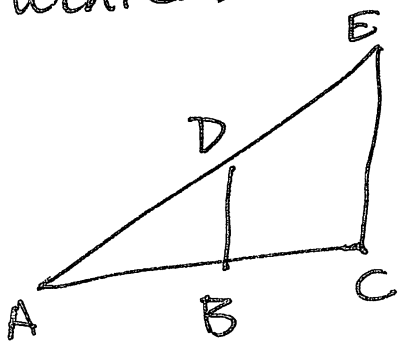
GIVEN $\overline{AC} \cong \overline{CB}$,

KNOW

$\triangle AXC \cong \triangle BXC$

(NOT SAME, BUT SAME)

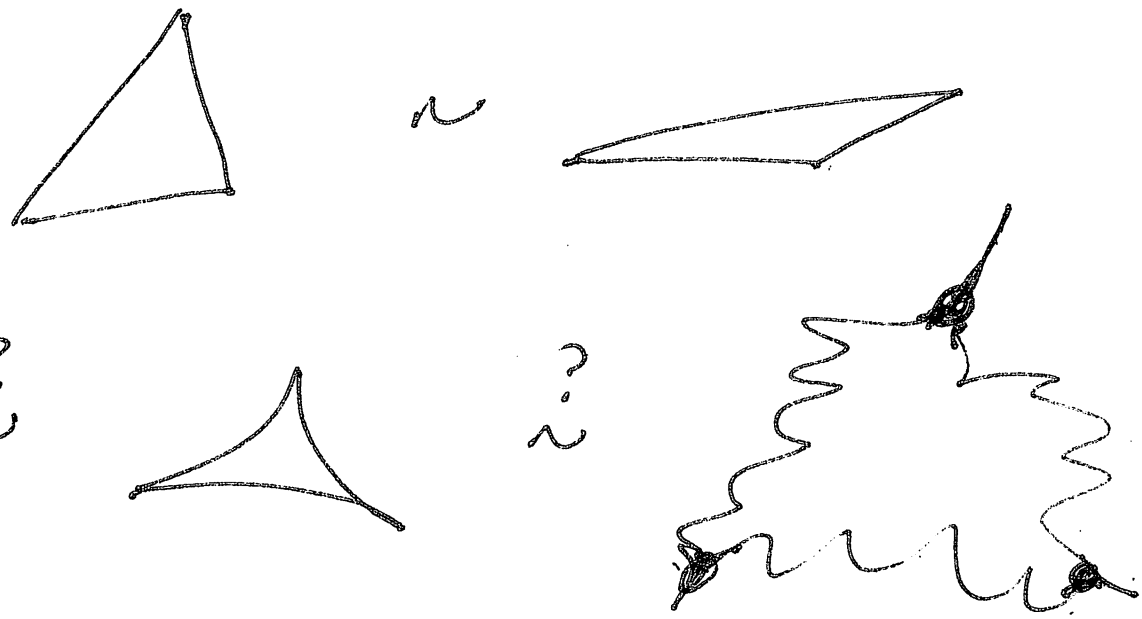
WEAKER IS SIMILARITY



$\triangle ABD \sim \triangle ACE$.

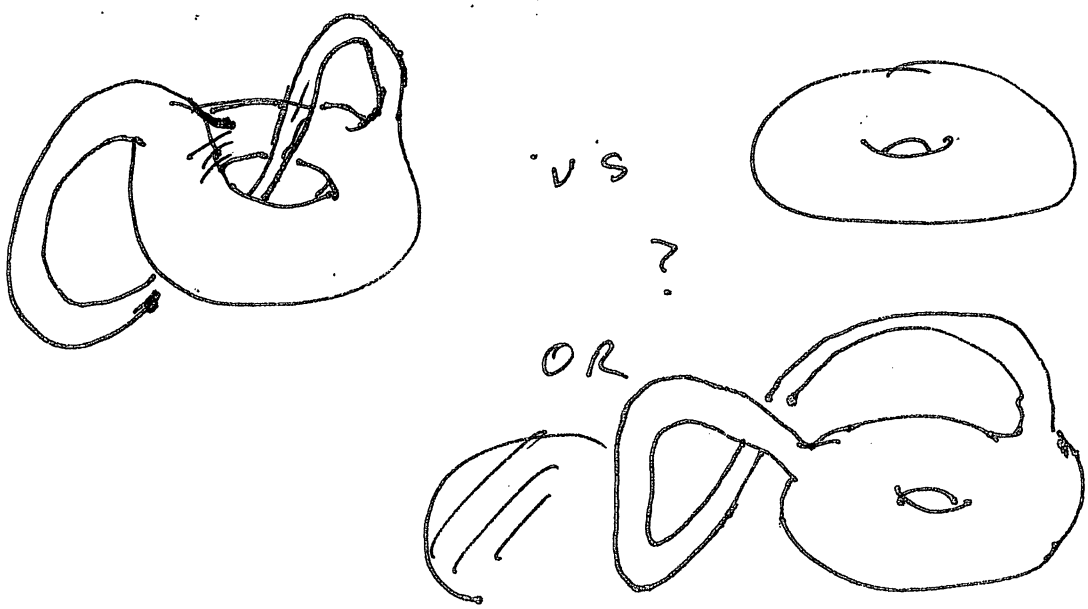
HERE "SHAPE" IS SAME, BUT SIZE IS DIFFERENT.

STILL WEAKER:



SO, WE HAVE TO DECIDE WHAT "THE SAME" MEANS.

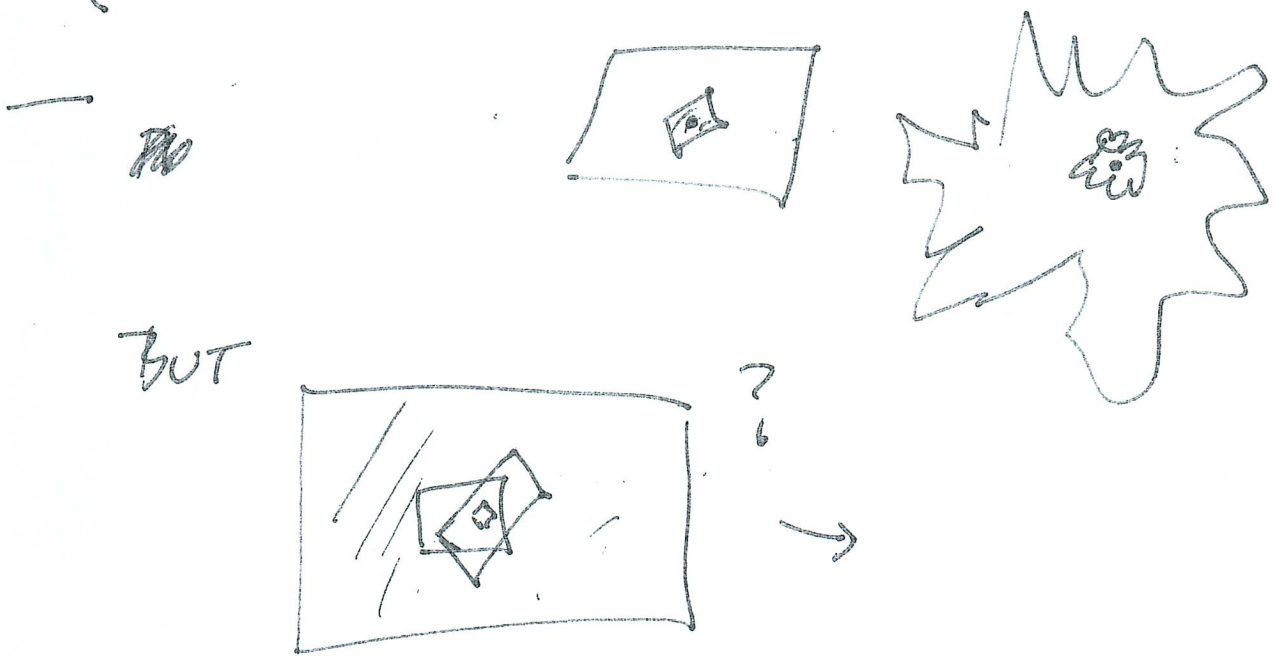
WHAT ABOUT



WHY DOES IT MATTER?

EXAMPLE: BROUER FIXED PT THM.

(THROW MAP ON FLOOR)



BUT

EQUIVALENCE RELATION $\tilde{\sim}$ IS

AN EQUIV. REL. IF

- SYMMETRIC ($\forall A,B$, IF $A \tilde{\sim} B$, THEN $B \tilde{\sim} A$)
- REFLEXIVE ($\forall A$, $A \tilde{\sim} A$ ALWAYS)
- TRANSITIVE ($\forall A,B,C$, IF $A \tilde{\sim} B$ & $B \tilde{\sim} C$ THEN $A \tilde{\sim} C$).

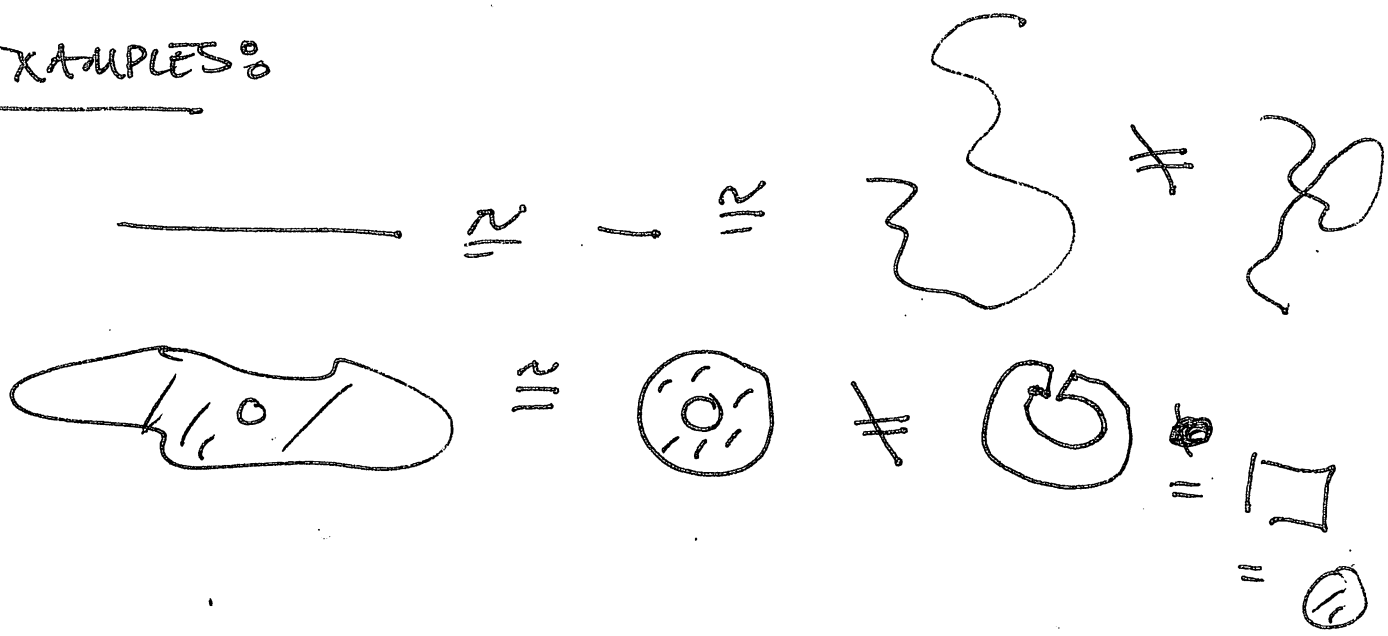
IN GEOMETRY,

OBJECT $A \cong B$ IF $\exists$ A RIGID MOTION (ISOMETRY) m SO THAT $m(A) = B$

TOPOLOGY IF WE CAN CONTINUOUSLY DEFORM A TO B (AND B BACK TO A)

IE, $\exists$ $f: A \to B$ W/ $f^{-1}: B \to A$ W/ f, f^{-1} CONTINUOUS.

EXAMPLES:



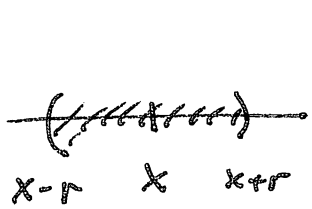
WE WILL FOCUS ON SUBSETS OF $\mathbb{R}^n$, i.e. POINTS w/ COORDS $\in \mathbb{R}^n = \{x = (x_1, x_2, \dots, x_n) \mid x_i \in \mathbb{R}\}$

IN $\mathbb{R}^1$, WE HAVE A NOTION OF DISTANCE

$$\|\vec{x} - \vec{y}\| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2}$$

THEN $D_r(x) = \{y \mid \|x - y\| < r\}$

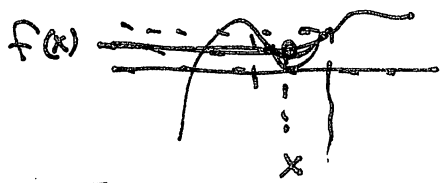
$= D^1(x; r)$



DEF: A (DISC) NEIGHBORHOOD OF x IN $\mathbb{R}^n$ IS $D_r(x)$ FOR SOME r.

WHAT DOES CONTINUOUS MEAN?

f CONT AT x:



$\forall \varepsilon > 0$, $\exists \delta > 0$ SO THAT

$\|x - y\| < \delta \Rightarrow \|f(x) - f(y)\| < \varepsilon$

BUT THIS IS ANNOYING: WE WANT TO FORGET ABOUT ACTUAL DISTANCE. SO WHAT IS THIS REALLY SAYING?