§**1.3#30** Consider the following geometry:

Undefined Terms:

Points: Points are interpreted as members of a set P given by

$$P = \{(x, y) \mid x, y \in \{0, 1\}\}$$

So, for example, the ordered pair $(0, 1)$ represents a "point" in this geometry, but the ordered pairs $(1, 2)$ and $(3, 4)$ do not.

Lines: Similarly, a line will also be interpreted as a member of a set L, given by

$$L = \{(x, y) \mid ax + by = c\}$$

with a, b, and $c \in \{0, 1\}$, but a and b are not both zero.
The operations of addition and multiplication will be computed using arithmetic modulo 2.

So, for example, if $a = 1$ and $b = 1$ and $c = 0$, the equation $ax + by = c$ becomes $1x + 1y = 0$ and represents a "line", which we will call ℓ_1, containing all the "points" that satisfy it. Therefore, the "point" $(0, 0)$ is on ℓ_1, but $(0, 1)$ is not. (Note that $(1, 1)$ is on ℓ_1 however, since $1 + 1 = 0$ when computed modulo 2.)

(a) How many points exist in this geometry? List them all and name them P_1 through P_n, where n is the number of points in the geometry.

(b) How many lines are there in this geometry?

(c) List the equations that determine the lines, and label them ℓ_1 through ℓ_m, where m is the number of lines in the geometry.

(d) Construct a model, concrete or abstract (or both) to show the consistency of the geometry

(e) Is this geometry isomorphic to any geometry we have seen before? If so, name the geometry and construct the isomorphism that associates the undefined terms of each. If not, explain how you can be sure.