Mat360 HW9 Solutions

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EX6.4.5
Proof:
It suffices to show $\angle ABP = \angle CBP$.
Now, let us forget the fifth postulate.
Because $AP = AP$, $\angle PAB = \angle PAC$,and p_1, p_3 are perpendicular to AB, AC.
So,by AAS,we have $p_1 = p_3$.Similarly,$p_2 = p_3$.
So $p_1 = p_2 = p_3$.

Now,wlog, let us assume that $\angle ABP > \angle CBP$.
Because p_1, p_2 are perpendicular to AB, BC.
By AAS and $\angle ABP > \angle CBP$,we have $p_1 > p_2$.It is impossible!
And we can also disprove the hypothesis $\angle ABP < \angle CBP$ by same argument.
So $\angle ABP > \angle CBP$,so BP is the bisector of $\angle ABC$.

That's O.K.!

EX6.4.7
Proof:
Step 1:It is easy to show that we can place $\triangle DEF$ inside $\triangle ABC$ with the help of in-center.

Step 2:From Step 1,we can know that the area of $\triangle DEF$ is smaller than $\triangle ABC$.
So the defect of $\triangle DEF$ is smaller than $\triangle ABC$.
So each interior angle of $\triangle DEF$ is smaller than that of $\triangle ABC$.

That's O.K.!

EX6.4.16
Proof:

Of course,the defect of each triangle in Euclidean Geometry is 0.
But there are a pair of triangles not congruent to each other.

$EX6.5.11$
(i)$Area(\triangle) = C \times (defect) = c \times (180^\circ - anglesum)$
Here the sum is $55^\circ \times 3 = 165^\circ$.
So $defect = 15^\circ$.
So $Area = c \times 15^\circ$.
(c is a positive constant).

(ii)From (i),it is easy to know that the area is $c \times (180 - 3x)$(with the measure of degree. c is a constant.)

$EX6.5.13$
(a)Proof:
$\triangle ADE$ is larger.

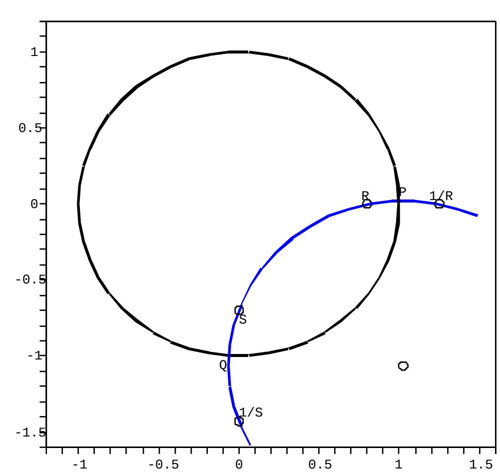
Because triangle is convex.
So $\triangle ABC$ is lying in $\triangle ABD$(not inside).

(b) Proof:
Obviously,$S(BCED) + S(\triangle ABC) = S(\triangle ADE)$,so
$S(BCED) = S(\triangle ADE) - S(\triangle ABC) = k_2^\circ - k_1^\circ$.

Remark:I think it is not elegant to compute the defect!

$EX6.6.12$
Solution:

In order to calculate the hyperbolic distance between $R = (0.8, 0)$ and $S = (0, -0.7)$, we need to find where the hyperbolic line $\overline{RS}$ "ends", that is, the points P and Q where it intersects the unit circle in the Euclidean plane (recall that our model of the hyperbolic plane is the interior of this circle). So, first we must find the line, and then discover where it intersects the unit circle.
The hyperbolic line also contains (in the Euclidean plane) the points $R' = (1/0.8, 0)$ and $S' = (0, -1/0.7)$ which are the reflections of R and S through the circle. This makes it easy to find the center, which lies on the perpendicular bisectors of both $\overline{RR'}$ and $\overline{SS'}$.



Thus, the center is

$$C = \left(\frac{0.8 + 1/0.8}{2}, \frac{-0.7 - 1/0.7}{2}\right) = \left(\frac{41}{40}, \frac{-149}{140}\right) \approx (1.025, -1.064285714)$$

and the radius is the distance from any of the points R, R', S, or S' to C, which is

$$r = \frac{\sqrt{92773}}{280} \approx 1.0878093.$$

Now our first task is to locate where this circle (hyperbolic line) intersects the unit circle $x^2 + y^2 = 1$, so we solve the resulting pair of simultaneous equations for x and y:

$$x^2 + y^2 = 1 \qquad \left(x - \frac{41}{40}\right)^2 + \left(y + \frac{149}{140}\right)^2 = \frac{92773}{78400}$$

which gives the pair of solutions

$$P = \left(\frac{80360 + 298\sqrt{92773}}{171173}, \frac{-83440 + 287\sqrt{92773}}{171173}\right) \approx (0.9997301463, 0.0232300403)$$

and

$$Q = \left(\frac{80360 - 298\sqrt{92773}}{171173}, \frac{-83440 - 287\sqrt{92773}}{171173}\right) \approx (-0.0607970201, -0.9981501503)$$

Now we are ready to use the formula:

$$dist(R, S) = \left|\ln\left(\frac{|SP|}{|RP|} \cdot \frac{|RQ|}{|SQ|}\right)\right|$$

which is approximately

$$\left|\ln\left(\frac{1.233905206}{0.2010765180} \cdot \frac{1.318057371}{0.3042857042}\right)\right| \approx |\ln(26.58111569)| \approx 3.280201027$$

$EX6.8.12$
Proof:
Because,we can divide any triangle into two right triangles.So,
$S(original\triangle) + 180° = S(right\triangle 1) + S(right\triangle 2) > 2 \times 180°$.
So we can deduce the inequality in this theorem.