MAT 360
Home Work 8

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April 3, 2004

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Problem6.3.7
Proof:
We denote these two lines as l and m,the common perpendicular is AB , here A in l,B in m. For any other segment CD, C in l,D in m.We try our best to let CD to be shorter than AB. So we can assume CD can not intersect AB.
Second,we can assume that CD is perpendicular to either l or m or both.
Then $ABCD$ is a *Lambert* quadrilateral,so by *Theorem*6.3.4,we can assume that $\angle ACD$ is acute.
So by *Theorem*6.3.5, we know that the length of CD is larger than that of its opposite side,AB.
(In the argument above,we assume that A, B, C, D are distinct points.If $C = D$,the case will be very easy.)

Problem6.3.20
Proof:
" $\Longrightarrow$ " :We denote these two lines as l and m,these two equidistance are AB and CD , here A and C in l,B and D in m.
Obviously,we can assume that $\angle CDB$ and $\angle ABD$ are both right angles.And AB does not intersect CD.
Let M, N be mid-points of AC and BD.Then connect M, N.
By Theorem 3.6.4,MN is perpendicular to both l and m.
" $\Longleftarrow$ ":We assume MN be the common perpendicular of l and m.Here M in l,N in m.
Let $AM = MC$,here A, C are on l.($A \neq C$),and AB, CD are perpendicular to m at B, D.
So by SAS,$AN = CN$ and $\angle ANM = \angle CNM$. Because MN is perpendicular to m.So we have that $\angle ANB = \angle CND$.
So by AAS, $\triangle ABN \cong \triangle CDN$,so $AB = CD$.
O.K.!