Answer For HW 6 — Page 1

Sec 4.3

Problem 4:

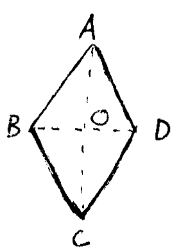
Definition of Rhombus. There are many ways; I pick two.

I: A quadrilateral, whose four sides are congruent.

II: A quadrilateral, whose diagnals are perpendicular to each other, and bisect each other.

Personally, II is better.

Proof of Th 4.3.6.




Denote the intersect of AC and BD as O.

So Area(Rhombus) = Area($\Delta ABO$) + Area($\Delta AOD$) + Area($\Delta COB$) + Area($COD$). (*)

Because diagnals are perpendicular to each other, bisecting each other.

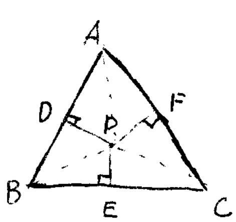
So Areas of there four little triangles in (*) are same.

so Area(Rhombus) = $4 \cdot$ Area(any little triangle),

Area(little triangle) = $\frac{1}{2}(BO)\cdot(AO) = \frac{1}{8}(AC)(BD)$

so Area(Rhombus) = $4\cdot(\frac{1}{8}(AC)(BD)) = \frac{1}{2}(AC)(BD)$.

Problem 13:




Proof: Area($\Delta ABC$) = Area($\Delta ABP$) + Area($\Delta BPC$) + Area($\Delta CPA$)

$= \frac{1}{2}[(AB)(PD) + (BC)(PE) + (AC)(PF)]$

$= \frac{1}{2}$(side length)$(PD + PE + PF)$

Other hand, Area($\Delta ABC$) = $\frac{1}{2}$(side length)$\cdot$(height) $\therefore$ By canceling...

Problem 17.

Proof: Connect the center (of the inscribed circle) with verticles of this regular n-gon. So, this regular n-gon is devided into n little triangles. We have:

$$\begin{cases} (*) \ \text{Area (this regular n-gon)} = \sum_{\text{little } \Delta} \frac{1}{2} (\text{Base})(\text{height}) \\ (**) \ (\text{height}) = (\text{radius}), \quad \sum_{\text{little } \Delta} \text{Base} = (\text{Perimeter of this n-gon}) \end{cases}$$

So (*), (**) implies that $\text{Area (n-gon, regular)} = \frac{1}{2}(\text{radius}) \cdot (\text{Perimeter})$
$= \frac{1}{2} a P$.

Remark: the existence of the inscribed circle of arbitany n-gon does not hold. (Why?) It holds only for admissive n-gon. Definitely, regular n-gon is "Best".

Problem 22.

Sol: Here $s = 12$,

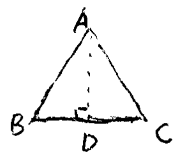
so $A^2 = 12(12-5)(12-9)(12-10) = 504$,

so $A = \sqrt{504} = 2 \cdot \sqrt{126} = 6 \cdot \sqrt{14} = 22.45$.

Answer for HW6 — Page 3

Sec 4.4

13.

Sol: 

As in figure rightside, draw a bisector of $\angle BAC$, which intersects $BC$ on $D$.

since $AB = BC$, $AD = AD$, $\angle BAD = \angle DAC$

so by SAS, $\Delta ABD$ is congruent to $ADC$

$\therefore$ We have facts that, $\begin{cases} BD = DC = \frac{s}{2} \\ \angle ADB = \angle ADC = 90^\circ \end{cases}$

$\therefore$ Apply Pythagorean Theorem to $\Delta ADB$, we have

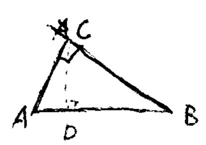
$(BD)^2 + (AD)^2 = (AB)^2$

$\therefore (\frac{1}{2}s)^2 + (AD)^2 = s^2 \Longrightarrow AD = \frac{\sqrt{3}}{2}s$

$\therefore$ Area$(\Delta ABC) = \frac{1}{2}(BC)(AD) = \frac{1}{2}s \cdot \frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2$

$\therefore$ Height $= \frac{\sqrt{3}}{2}s$, Area$(\Delta ABC) = \frac{\sqrt{3}}{4}s^2$

15. There are two ways we have:

Way 1: 

(What we need to show is $(CD)^2 = (AD)\cdot(BD)$)

$\because \angle CAD = \angle BCD$, $\angle CDA = \angle CDB = 90^\circ$, $CD = CD$

$\therefore \Delta CAD \cong \Delta CDB$ $\therefore \frac{CD}{AD} = \frac{BD}{CD}$ $\therefore (CD)^2 = (AD)(BD)$

Way 2: (with same figure in Way 1)

let $BD = y$, $AD = x$.

Apply Pythagorean Theorems to $\Delta CDA$, $\Delta CDB$, $\Delta ACB$, respectively

We have (add all 3 identities)

$$\begin{cases} x^2 + (CD)^2 = (AC)^2 \\ y^2 + (CD)^2 = (BC)^2 \\ (AC)^2 + (BC)^2 = (x+y)^2 \end{cases} \Longrightarrow x^2 + y^2 + 2(CD)^2 = (x+y)^2 \Longrightarrow (CD)^2 = xy$$

Problem 22.

Sol: Let $BC = 1$, $AB = \lambda < 1$,

so $\begin{cases} \dfrac{CF}{CD} = \dfrac{AB}{BC} \\ CD = AB = \lambda \\ CF = 1 - \lambda \end{cases} \Longrightarrow \dfrac{1-\lambda}{\lambda} = \lambda \Longrightarrow \lambda^2 = 1 - \lambda \Longrightarrow \lambda = \dfrac{\pm\sqrt{5} - 1}{2}$

of course, $\lambda$ must be positive, so $\lambda = \dfrac{\sqrt{5} - 1}{2}$