Midterm Exam MAT 360, Spring 2004

PRINT your Name: *(2 pts)*

problem	0	1	2	3	4	5	6	Total
possible	2	18	20*	20*	20*	20*	20*	100
score								

Directions: There are 6 problems on 3 pieces of paper (printed front and back) in this exam, with an additional blank page at the back. Do all of your work in this exam booklet, and cross out any work that should be ignored. You may not refer to any notes, texts, or friends you brought with you. Feel free to refer to notes you left at home, provided you can do this without leaving the room or speaking to anyone. Telepathic communication with people at least 100 yards from the classroom is also permitted, provided you can recreate it later in a scientifically verifiable study.

1. *(18 points)* For each of the following statements, indicate which of the following alternatives holds by circling the appropriate word. The choices are mutually exclusive, so just pick one.

Neutral The statement is always true in neutral geometry.

Euclidean The statement is always true in Euclidean geometry, but not in hyperbolic geometry.

Hyperbolic The statement is always true in hyperbolic geometry, but not in Euclidean geometry.

nope In both Euclidean and hyperbolic geometries, there are instances when the statement fails to hold.

a. If $|AB| = |PQ|$, $|BC| = |QR|$, and $|AC| = |PR|$, then triangles ABC and PQR are congruent.

Neutral **Euclidean** **Hyperbolic** **nope**

b. If $\triangle ABC$ and $\triangle PQR$ have $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$, and $|AB| < |PQ|$, then the triangles are similar.

Neutral **Euclidean** **Hyperbolic** **nope**

c. If line t is tranverse to parallel lines, the measures of the resulting alternate interior angles are equal.

Neutral **Euclidean** **Hyperbolic** **nope**

d. There is a triangle with the sum of the measures of its angles being 160 degrees.

Neutral **Euclidean** **Hyperbolic** **nope**

e. There is a triangle with the sum of the measures of its angles being 190 degrees.

Neutral **Euclidean** **Hyperbolic** **nope**

f. If line t is perpendicular to both line ℓ and line m, then lines ℓ and m are parallel.

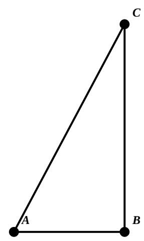
Neutral **Euclidean** **Hyperbolic** **nope**

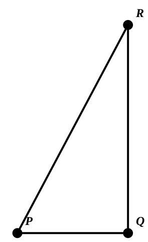
Do any four of problems 2-6. Cross out the one you don't want graded.

2. *(20 points)* In Euclidean geometry, we proved that if two triangles have three angles of equal measure, then the triangles are similar (The AAA similarity condition). Prove that whenever a pair of triangles have two corresponding angles of equal measure, the two triangles are similar.

Do any four of problems 2-6. Cross out the one you don't want graded.

3. *(20 points)* In neutral geometry, prove the hypotenuse-leg congruence condition: If two right triangles ABC and PQR have hypotenuses of equal length, and a leg of one is congruent to a leg of the other, then $\triangle ABC$ is congruent to $\triangle PQR$. (Hint: constructing an isosceles triangle might be helpful.)

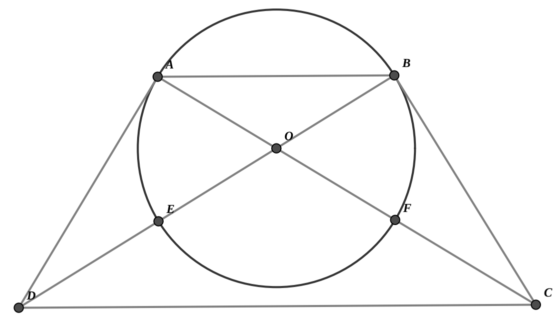




Do any four of problems 2-6. Cross out the one you don't want graded.

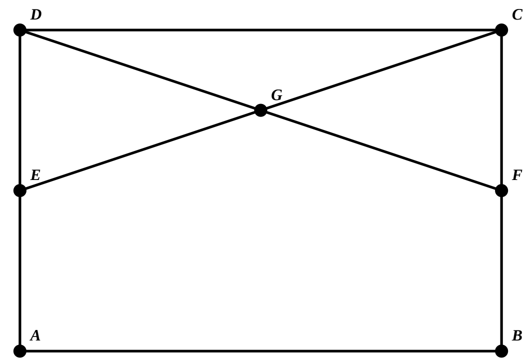
4. *(20 points)* In Euclidean geometry, the figure below has a circle with center O and radius OA. Lines BC and AD are tangent to the circle at B and A, with secant lines $DEOB$ and $CFOA$.

Prove that quadrilateral ABCD is an isosceles trapezoid, that is, that $|AD| = |BC|$ and that $\overline{AB}$ is parallel to $\overline{DC}$.



Do any four of problems 2-6. Cross out the one you don't want graded.

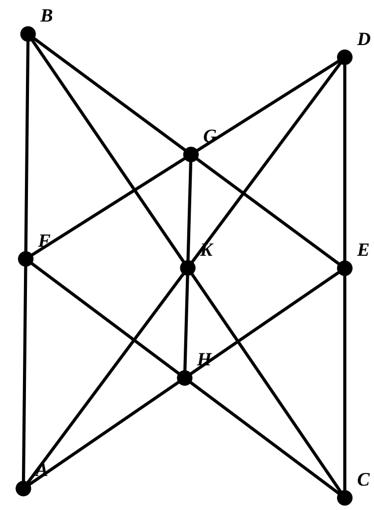
5. *(20 points)* In neutral geometry, let $\square ABCD$ be a Saccheri quadrilateral, with right angles at A and B and $|AD| = |BC|$. Let E and F be the midpoints of AD and BC, respectively. Prove that $|EC| = |DF|$.



Do any four of problems 2-6. Cross out the one you don't want graded.

6. *(20 points)* Consider the *Geometry of Pappus*, with the following axioms:

1. There exists at least one line.
2. Every line has exactly three points on it.
3. Not all points are on the same line.
4. If a point P is not on a line ℓ, then there is exactly one point Q on ℓ such that no line contains both P and Q.
5. With the exception of the above axiom, if P and Q are distinct points, then exactly one line contains them both.



A model for the Geometry of Pappus is shown at right.

Prove that every point is on at least three lines. Be careful to use the axioms in your proof, and not rely on the model. (In fact, every point is on exactly three lines, but you don't have to prove that.)

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You can make it even less blank if you find that helpful.