16. (*expires 3/6*) The set of twelve points

$$\begin{array}{cccc} (-2.256, 0.879) & (-1.764, 5.800) & (-0.684, -0.854) & (-0.776, 6.750) \\ (3.718, 7.394) & (0.081, -1.315) & (-2.357, 4.534) & (6.485, 4.021) \\ (6.518, 3.999) & (2.818, 7.689) & (1.788, -1.668) & (-2.720, 2.719) \end{array}$$

approximate a circle with an unknown radius and center at $(2, 3)$. What is the "best" value for r corresponding to this data? Explain your answer. Plot the resulting circle and the data points on the same graph.

(Note that if you fit the data using the method described in section 6 of the notes, you'll get a somewhat different radius. Using an unknown center gives one at approximately $(1.970, 3.002)$ due to the noise in the data.)

If you don't want to retype the points, you can load them from the file circdata.txt on the problems area on the class web page, which defines a list called `data`.

NOTE: Neither of the next two problems intrinsically involve `Maple`, except as a word processor to write your solution (although you can use it to help with calculations if you want). If you prefer, you are welcome to turn in your solution as a PDF file, which you can produce with a word processor or by scanning a hand-written version. A worksheet is slightly preferred.

17. (*expires 3/6*) Following Section 6 (Fitting a Circle) of the notes, prove that if we describe the circle of center (a, b) and radius r using the parameters (a, b, k), with $k = a^2 + b^2 - r^2$, rather than the more natural parameters (a, b, r), then the function $H(a, b, k) = E(a, b, \sqrt{a^2 + b^2 - k})$ is quadratic in a, b and k. What does this imply about the number of critical points?

18. (*expires 3/6*) With reference to the previous problem, show that for $r > 0$, the transformation $(a, b, r) \mapsto (a, b, k)$ is a valid change of variables, that is, it is a diffeomorphism (a one-to-one function with continuous nonzero derivatives). This should help you prove that $E(a, b, r)$ has only one "physical" critical point, which is a minimum, and is mapped, through the transformation, into the unique critical point of $H(a, b, k)$.