```
> A := <<1, 2>|<2, 5>>;
```

$$A := \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \quad (1)$$

```
> with(LinearAlgebra) :
> Determinant(A);
```

$$1 \quad (2)$$

```
> Trace(A);
```

$$6 \quad (3)$$

```
> Eigenvalues(A); evalf(%);
```

$$\begin{bmatrix} 3+2\sqrt{2} \\ 3-2\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 5.828427124 \\ 0.171572876 \end{bmatrix} \quad (4)$$

```
> B := <<1, 2>|<4, 5>>;
```

$$B := \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} \quad (5)$$

```
> LinearAlgebra:-Eigenvalues( (5) )
```

$$\begin{bmatrix} 3+2\sqrt{3} \\ 3-2\sqrt{3} \end{bmatrix} \quad (6)$$

```
> evalf(%);
```

$$\begin{bmatrix} 6.464101616 \\ -0.464101616 \end{bmatrix} \quad (7)$$

From TrDet.mw in problems area of website.

Make the movie

```
> with(LinearAlgebra):
> with(DEtools):
> with(plots): with(plottools):

> AvR:=d-> <<1+sqrt(1-d)/2|1>,<0|1-sqrt(1-d)>>;
  AvL:=d-> <<-1+sqrt(1-d)/2|1>,<0|-1-sqrt(1-d)/2>>;
  AhT:=t-> <<t/2|1>,<-1+t^2/4|t/2>>;
  AhB:=t-> <<t/2+evalf(1/sqrt(2))|1>,<1-t^2/4|t/2-evalf(1/(sqrt
  (2))+(t+2)/(4*sqrt(2)))>>;
```

$$AvR := d \rightarrow \left\langle \left\langle 1 + \frac{1}{2}\sqrt{1-d} \middle| 1 \right\rangle, \left\langle 0 \middle| 1 - \sqrt{1-d} \right\rangle \right\rangle$$

$$AvL := d \rightarrow \left\langle \left\langle -1 + \frac{1}{2}\sqrt{1-d} \middle| 1 \right\rangle, \left\langle 0 \middle| -1 - \frac{1}{2}\sqrt{1-d} \right\rangle \right\rangle$$

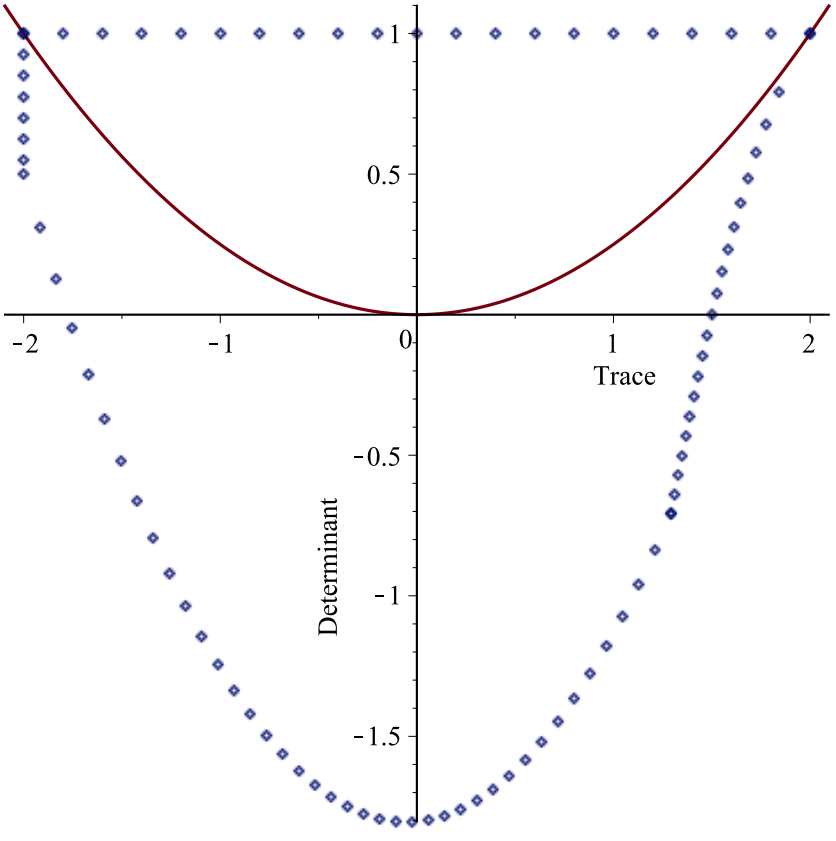
$$AhT := t \to \left\langle \left\langle \frac{1}{2}\ t \middle| 1 \right\rangle, \left\langle -1 + \frac{1}{4}\ t^2 \middle| \frac{1}{2}\ t \right\rangle \right\rangle$$

$$AhB := t \to \left\langle \left\langle \frac{1}{2}\ t + evalf\left(\frac{1}{\sqrt{2}} \right) \middle| 1 \right\rangle, \left\langle 1 - \frac{1}{4}\ t^2 \middle| \frac{1}{2}\ t - evalf\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\ \frac{t+2}{\sqrt{2}} \right) \right\rangle \right\rangle \qquad \textbf{(1.1)}$$

```
> AtoSys:=proc(A)
    return([diff(x(t),t)=A[1,1]*x(t) + A[1,2]*y(t), diff(y(t),t)=
  A[2,1]*x(t) + A[2,2]*y(t)]);
  end:
> EvInits:=proc(A)
    local EvM,t,d;
    t:=A[1,1]+A[2,2];
    d:=A[1,1]*A[2,2] - A[1,2]*A[2,1];
    if (evalf(d-t^2/4>0)) then
       return();
     fi:
    EvM:=map(Re,Eigenvectors(A)[2]);
    return( [x(0)=EvM[1,1]/2,y(0)=EvM[2,1]/2], [x(0)=-EvM[1,1]/2,
  y(0)=-EvM[2,1]/2],
          [x(0)=EvM[1,2]/2,y(0)=EvM[2,2]/2], [x(0)=-EvM[1,2]/2,
  y(0)=-EvM[2,2]/2]);
  end:
> Icirc:=proc()
    local t,ini;
    ini:=[];
    for t from 0 to 2 by .12 do
      ini:=[op(ini),[x(0)=evalf(1.5*cos(t*Pi)),y(0)=1.5*evalf(sin
  (t*Pi))]];
    od;
    return(op(ini));
  end:
  Ihline:=proc()
    local t,ini;
    ini:=[];
    for t from -2 to 2 by .25 do
      ini:=[op(ini),[x(0)=t,y(0)=0]];
    od;
    return(op(ini));
  end:
> EvLine:=proc(A)
    local EvM,t,d;
    t:=A[1,1]+A[2,2];
    d:=A[1,1]*A[2,2] - A[1,2]*A[2,1];
    if (evalf(d-t^2/4>0)) then
       return();
     fi:
    EvM:=map(Re,Eigenvectors(A)[2]);
    return( [line( [3*EvM[1,1],3*EvM[2,1]],[-3*EvM[1,1],-3*EvM[2,
  1]], color=blue, thickness=3),
          line( [3*EvM[1,2],3*EvM[2,2]], [-3*EvM[1,2],-3*EvM[2,
  2]], color=blue, thickness=3)]);

  end:
```

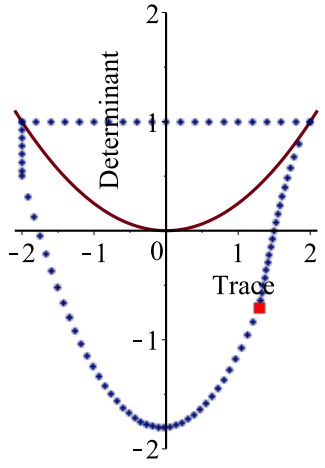
```
> ptsVR:=seq([Trace(AvR(det)),Determinant(AvR(det))], det=-1..1,
  .1):

> ptsVL:=seq([Trace(AvL(det)),Determinant(AvL(det))], det=1..-1,
  -.3):
> ptsHT:=seq([Trace(AhT(tr)), Determinant(AhT(tr))],  tr=2..-2,
  -.2):
> ptsHB:=seq([Trace(AhB(tr)), Determinant(AhB(tr))],  tr=-2..2,
  .1):
> bg:=plot([x^2/4,[ptsVR,ptsHT,ptsVL,ptsHB]],x=-2.1..2.1,style=
  [line,point],
     labels=["Trace","Determinant"],labeldirections=[horizontal,
  vertical]);
  bg;
```

$$bg := PLOT(\ldots)$$

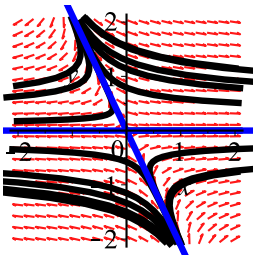



```
> PlotSpot:=(Tr,Det)->display([bg,plot([[Tr,Det]],x=-2..2,y=-2.
```

```
  .2,style=point,symbolsize=22,symbol=solidbox,color=red)]):
>
> vFrameR:=seq(
   display(Array([[
    PlotSpot(Trace(AvR(det)),Determinant(AvR(det))),
     display([
         DEplot(AtoSys(AvR(det)),[x(t),y(t)],t=-5..5,x=-2..2,y=
  -2..2,[Icirc()],linecolor=black),
         display(EvLine(AvR(det)),view=[-2..2,-2..2])
        ], scaling=constrained, caption=typeset("trace=",Trace
  (AvR(det)),", det=",Determinant(AvR(det))),
           title=typeset(lambda[1]=Eigenvalues(AvR(det))[1],"
   ",lambda[2]=Eigenvalues(AvR(det))[2]))
    ]])),
    det=-1..1,.1):
>
> vFrameL:=seq(
    display(Array([[
     PlotSpot(Trace(AvL(det)),Determinant(AvL(det))),
      display([
         DEplot(AtoSys(AvL(det)),[x(t),y(t)],t=-5..5,x=-2..2,y=
  -2..2,[Icirc()],linecolor=black),
         display(EvLine(AvL(det)),view=[-2..2,-2..2])
        ], scaling=constrained, caption=typeset("trace=",Trace
  (AvL(det)),", det=",Determinant(AvL(det))),
           title=typeset(lambda[1]=Eigenvalues(AvL(det))[1],"
   ",lambda[2]=Eigenvalues(AvL(det))[2]))
    ]])),
    det=1..-1, -.3):
> hFrameT:=seq(
    display(Array([[
     PlotSpot(Trace(AhT(tr)),Determinant(AhT(tr))),
       display(
         DEplot(AtoSys(AhT(tr)),[x(t),y(t)],t=-5..5,x=-2..2,y=
  -2..2,[Icirc(),Ihline()],linecolor=black)
        , scaling=constrained, caption=typeset("trace=",Trace
  (AhT(tr)),", det=",Determinant(AhT(tr))),
           title=typeset(lambda[1]=Eigenvalues(AhT(tr))[1],"
  ",lambda[2]=Eigenvalues(AhT(tr))[2]))
    ]])),
    tr=2..-2, -.2):
> hFrameB:=seq(
    display(Array([[
     PlotSpot(Trace(AhB(tr)),Determinant(AhB(tr))),
       display([
         DEplot(AtoSys(AhB(tr)),[x(t),y(t)],t=-5..5,x=-2..2,y=
  -2..2,[Icirc()],linecolor=black),
         display(EvLine(AhB(tr)),view=[-2..2,-2..2])
        ], scaling=constrained, caption=typeset("trace=",Trace
  (AhB(tr)),", det=",Determinant(AhB(tr))),
           title=typeset(lambda[1]=Eigenvalues(AhB(tr))[1],"
  ",lambda[2]=Eigenvalues(AhB(tr))[2]))
    ]])),
    tr=-2..2, .1):
> display([vFrameL,hFrameB, vFrameR,hFrameT],scaling=constrained,
  insequence=true);
```



$$\lambda_1 = 1 + \frac{1}{2}\sqrt{2} \quad \lambda_2 = 1 - \sqrt{2}$$



$$\text{trace}=2 - \frac{1}{2}\sqrt{2}, \ \det=$$

$$-\frac{1}{2}\left(2+\sqrt{2}\right)\left(-1+\sqrt{2}\right)$$

>