MAT 331 Project 3

Due December 13

NOTE: *Your project should be typed and presented as a paper which explains clearly each step you take. The expository part of this project should be written so that anyone, with general knowledge of mathematics, but not necessarily of fractals and fractal dimensions can understand your paper. You should explain the algorithm for every program that you used in the project. If you don't like Maple as a programming tool, you can use C/C++, Java or Perl for any problem in this assignment. If you decide to do that, submit electronically the source code which can be compiled and executed. However the source code that doesn't compile is worth 0 points. Comment your code.*

In this project you will work with different fractals and determine their dimensions.

Question 1: Recall that in the construction of von Koch curve each time we replace the middle third of an interval by the other two sides of an equilateral triangle. As a modification, we might toss a coin to decide whether to position the new part 'above' or 'below' the removed segment. After a few steps we should get a rather irregular looking curve which nevertheless retains certain of the characteristics of the von Koch curve.

Modify the construction of the von Koch curve that we discussed in class to draw the 'random' version of the curve. Give at least three different instances of such random construction.

Question 2: Show that a random von Koch curve in Problem 1 always has Hausdorff dimension

$$s = \frac{\log 4}{\log 3}.$$

Question 3: Let F consist of those numbers in $[0, 1]$ whose decimal expansion do not contain the digit 5. Find $\dim_B F$, showing that this box-counting dimension exists.

Question 4: Determine the Hausdorff and box-counting dimensions of the set

$$\left\{0, 1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots\right\}.$$

What properties of the dimensions does this example reveal?

Question 5: Describe the construction of the Sierpinski gasket G. Draw the gasket using Maple. Prove that

$$\dim_H G = \dim_B G = \frac{\log 3}{\log 2}.$$