MAT 331 Project 2

Due November 15

NOTE: *Your project should be typed and presented as a paper which explains clearly each step you take. Also explain the algorithm that you use to display Julia and Mandelbrot sets. The expository part of this project counts as much as the actual calculations, and should not be ignored. You should explain the algorithm for every program that you used in the project. If you don't like Maple as a programming tool, you can use C/C++, Java or Perl for any problem in this assignment. If you decide to do that, submit electronically the source code which can be compiled and executed. Comment your code.*

In this project you will work with the Julia and Mandelbrot set of a certain polynomial map of degree two in one complex variable.

Question 1: Consider the mapping

$$z_{n+1} = z_n^2 + c, \tag{1}$$

where the constant c can be found in the data file (see the class web page for details). Iterate $z_0 = 0$ for 500 times and list the last 20 points. What can you say about the orbit of this point? What are the other types of orbits that this map has? Explain and show examples.

Question 2: Plot the Julia set of the mapping $z_{n+1} = z_n^2 + c$. Use the same constant c as in Question 1. Determine the fixed points. Which of the fixed points are attractors? Also find the periodic points of period two and three for this map.

Question 3: Plot the Mandelbrot set for the map given by equation (1). Find a constant c in equation (1) such that the Julia set of the map is totally disconnected. Support you answer by an image of such Julia set. What can you say about the orbit of the point $z_0 = 0$ in this case?

Question 4: Determine the formulas for periodic points of periods one and two for the mapping $z_{n+1} = c - z_n^2$. (Your answer to this question will be a function of c.)

Question 5: Modify the algorithm used in Question 2 to visualize the Julia set of the map $f(z) = z^3 + \frac{z}{2}$. What can you say about this set?

Question 6 (Extra Credit): Determine the fixed points for the mapping

$$z_{n+1} = 2 + \frac{z_n e^{i|z_n|^2}}{10}.$$

You will find a file containing the data for this project in the file
/home/mat331/www/projects/data2.txt