MAT 319: MIDTERM 2

FRIDAY, NOV. 10

Your name:____
(please print)

This is a **takehome exam**. It is due on Monday, Nov. 13, in class. Late submissions are not accepted under any circumstances.

You are allowed to use the textbook, your notes, and previous homeworks and their solutions, but no other sources. Of course, no consultations with anyone — your fellow students, friends, tutors, etc. Any breaking of these rules will be considered cheating and referred to Academic Judiciary committee.

Since it is a takehome exam, I expect full and clearly written solutions. Unless a problem explicilty states "no explanation required", please try to write as detailed an explanation as possible. Explanations should be such that someone who does not know how to solve this problem (but knows all previous material) can follow your arguments and understand what you are doing. When using some result from the texbook, given precise reference, e.g. "by Theorem 3.5.8".

Answers without explanations will get very little partial credit!

Notation:
$\mathbb{Z}$ — integer numbers
$\mathbb{N}$ — positive integers
$\mathbb{R}$ — real numbers

There are 5 problems in this exam. Each problem is worth 10 pts. Good luck!

	1	2	3	4	5	**Total**
Grade						

1. Let a_n be a sequence such that the subsequences a_{2n} and a_{2n-1} are convergent: $\lim a_{2n} = A$, $\lim a_{2n-1} = B$. Show that the sequence a_n is convergent iff $A = B$.

2. Let the sequence A_n be defined by $a_1 = 1.5$, $a_{n+1} = \frac{1+a_n}{2a_n}$. Prove that a_n is convergent and find the limit. [Hint: prove first that for all n, $3/4 \leq a_n \leq 1.5$]

3. Let sequences a_n, b_n be such that $\lim b_n = \infty$ and there exist $M, N \in \mathbb{R}$, $M > 0$ such that for all n, $M \leq \frac{a_n}{b_n} \leq N$. Show that then $\lim a_n = \infty$.

4. For each of the following series, determine whether it is convergent (you do not have to find the sum).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n}{2^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n+(-1)^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n5^n}{n!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\varepsilon_n}{n\sqrt{n}}$,

where

$$\varepsilon_n = \begin{cases} 1, & n \text{ is prime} \\ -1, & \text{otherwise} \end{cases}$$

5. Prove from the definition that for any $c \geq 0$,

$$\lim_{x \to c} \sqrt[3]{x} = \sqrt[3]{c}$$