MAT319/320 First Midterm

October 8, 2008

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

There are 5 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Feel free to consult the Psychic Friends Network if you can do so telepathically. I'm not sure Dionne Warwick knows much analysis, and besides, they went bankrupt about 10 years ago. But go for it, if you wish.

You have an hour, more or less, to complete this exam.

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1. $\boxed{\text{10 points}}$ Give a careful and complete definition of what it means when we say "The limit of the sequence X is L."

2. $\boxed{\text{10 points}}$ Let A and B be bounded subsets of $\mathbb{R}$.

   (a) Prove that $A \cup B$ is a bounded subset of $\mathbb{R}$.

   (b) Prove that $\sup(A \cup B) = \sup(\sup A, \sup B)$.

3. (a) $\boxed{\text{10 points}}$ Prove that for all natural numbers n, we have $2^n \geq n+1$.
   You might find induction helpful.

   (b) Prove that for all natural numbers $n \geq 4$, we have $2^n \geq n^2$.
   Feel free to use the result from part a, even if you couldn't do it.

4. $\boxed{\text{10 points}}$ Consider the sequence whose n^{th} term $a_n = \left(\frac{-1}{2}\right)^n$. Prove, using the definition of the limit, that the limit of this sequence is 0.

5. $\boxed{\text{10 points}}$ Let $f : (0,1) \to \mathbb{R}$ have the property that $f(x) < x$ for all $x \in (0,1)$.

   (a) Prove that $\sup\limits_{x\in(0,1)} f(x) \leq 1$

   (b) Is it true that $\sup\limits_{x\in(0,1)} f(x) < 1$? Prove or give a counterexample.