MAT 307 FALL 2011 MIDTERM I EXAM

Name: **ID Number:**

Problem	1	2	3	4	5	6	Total
Score							

Directions: Each problem is 15 points; some problems are easier, some harder. Do all of your work on these pages and cross out any work that should be ignored. Use the backs of pages if needed, indicating where you've done this. Notes, books, calculators and discussions with others are not permitted.

1. Compute the gradient ∇f of the function $f(x, y) = xy$, and sketch the gradient vector field.

2. Consider the functions

$$f(u,v) = (u+v, u-v, u^2-v^2), \quad g(x,y,z) = x^2+y^2-z^2.$$

Find the derivatives of f, g and of the composition $g \circ f$.

3. The equation

$$y^3x - xz^2 + z^5 = 9$$

defines a surface S in $\mathbb{R}^3$.

a. Find a vector orthogonal (perpendicular) to S at the point $(-1, 3, 2)$ on S.

b. Near the point $(-1, 3, 2)$, the equation above defines z implicitly as a function of x, y, so $z = z(x, y)$ and S is the graph of the function $z(x, y)$. Compute the partial derivatives $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ at $(-1, 3, 2)$.

4. Find all critical points of the function

$$f(x, y) = 2x^2 - 4xy + y^2 + y^3$$

and determine whether each critical point is a local maximum, local minimum or a saddle point.

5. Find the extreme values (maximum and minimum) of $f(x, y) = xy$ on the ellipse $x^2 + 2y^2 = 1$.

6. Find the absolute max/min values of $f(x, y) = x^2 + xy + 2y^2$ on the domain $D = \{(x, y) : x^2 + 2y^2 \leq 1\}$.