1. Introduction

1.1. Physical vs. Ideal. What is a triangle? Is a triangle a physical object made up of 3 straight pieces of wood or metal or somesuch, joined at the corners, or is it an ideal object consisting of lines that have no width lying in a plane that has no thickness?

1.2. The idea of constructibility. Historically, all lengths and angles are somehow constructible. That is, they are abstract objects that, in some sense are capable of being realized as physical objects. We will take a somewhat different point of view; we will assume familiarity with real numbers, and with the correspondence between real numbers and points on a line.

1.3. Basic objects. The plane, lines, points, length and distance, angle measure.

1.4. Basic concepts.

- Points lie on lines.
- If two distinct points lie on a line, then the length of the line segment between these points is well defined.
- A point separates a line into two half-lines.
- Two distinct points on the same line separate it into a line segment, which has a length (the length is a positive number), and two half-lines.
- A line separates the plane into two half-planes, which are regions (in modern terms, a region is a connected open subset of the plane).
- Two distinct lines either meet at a point, or are disjoint, in which case they are parallel.

1.5. Basic axioms. These are the first few; a few more will follow. The reader should be aware that the numbering of the axioms, as well as the theorems, propositions, etc. is unique to these notes. It is also possible to have the same notion of planar geometry with a slightly different collection of axioms, but these are what we shall use.

Axiom 1. *Two distinct lines intersect in at most one point.*

Axiom 2. *Any two distinct points lie on a line.*

Axiom 3. *If two lines intersect in a point, they separate the plane into 4 regions, called sectors, and they define an angle in each of these sectors; the sum of the measures of any two adjacent angles is π.*

Axiom 4. *If two lines do not intersect, they divide the plane into three regions, with exactly one of them, the one between the two lines, having both lines on its boundary.*

Exercise 1.1: Using the above axioms, show that given any two distinct points, there is exactly one line that contains them both.

1.6. Basic notations. If A and B are distinct points, then the (unique) line on which they lie is denoted by AB. The line segment between A and B is also denoted by AB; this should cause no confusion. The length of the line segment AB is denoted by $|AB|$.

If AB and AC are distinct lines or line segments, the angle between them is denoted by $\angle BAC$, and its measure is denoted by $m\angle BAC$. We may use $\angle A$ to denote an angle which has the point A at its vertex, if it is clear from the context which angle is being referred to.

It is also not unusual to use Greek letters such as α, β, θ, φ, etc. to denote both angles and their measures.

Theorem 1.1 (Vertical angles)**.** *Vertical angles are equal. That is, if two distinct lines intersect at a point, the measure of the angles of any two non-adjacent sectors is equal.*

Exercise 1.2: Prove Thm. 1.1. You may use any of the axioms above, along with logical axioms and results for real numbers (since angle measure is a real number).

1.7. Basic constructions. (more will follow)

- Any line segment can be extended in either direction, or in both directions.
- If $|AB| < |CD|$, then there is a point E on the line CD, where E lies between C and D, so that $|AB| = |CE|$ (see also Axiom **??**).
- If A and B are points on the line k, and we are given an angle $\angle CDE$, where $m\angle CDE \neq \pi$, then we can construct points F and G, one on each side of the line $k = AB$, so that $m\angle BAF = m\angle BAG = m\angle CDE$. (see also Axiom **??**).