MATH 141 First Midterm

October 18, 2010

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	Total
Points:	20	15	15	20	20	90
Score:						

There are 5 problems on 6 pages in this exam (not counting the cover sheet, but counting the blank page at the end). Make sure that you have them all.

You may use a calculator if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** If you managed to convince Michelle to show up for the exam, you may ask her for help.

You have about 59 minutes and 47 seconds to complete this exam.

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

$\boxed{\text{5 pts.}}$ 1. (a) Give a complete and careful definition of the following statement:
"The sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ converges to the limit L."

$\boxed{\text{5 pts.}}$ (b) Let $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$. Give a complete definition of the following statement:
"The function $f(x)$ is continuous at $x = a$."

$\boxed{\text{5 pts.}}$ (c) If A is a subset of the real numbers, give a complete and careful definition of the following statement: $\sup A = \alpha$.

$\boxed{\text{5 pts.}}$ (d) State the Intermediate Value Theorem for a function $f : \mathbb{R} \to \mathbb{R}$.

2. Consider the sequence

$$\{r_n\} = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \ldots$$

5 pts.

(a) If this sequence has any accumulation points, list one of them. If there are none, write "none". In either case, give some justification of your answer (it needn't be a proof, just a reason why your answer is reasonable.)

10 pts.

(b) Describe *all* accumulation points of the sequence above. As before, if there are none, just write "none". Give a justification of your answer.

$\boxed{\text{15 pts.}}$ 3. Prove that for any integer $n \geq 1$, $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = (1 + 2 + 3 + 4 + \ldots + n)^2$.

The formulae $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ might or might not be helpful. Induction could be your friend, too.

$\boxed{\text{15 pts.}}$ 4. (a) Let f and g be functions defined on all of $\mathbb{R}$, with f strictly increasing and g strictly decreasing. Prove that there is *at most* one point $a \in \mathbb{R}$ with $f(a) = g(a)$. Do not assume f or g are continuous.
(Hint: what goes wrong if there are two distinct points where f and g are equal?)

$\boxed{\text{5 pts.}}$ (b) Give an example of a pair of continuous functions f and g, defined on all of $\mathbb{R}$, with f strictly increasing and g strictly decreasing and so that f and g are never equal.

5. For each of the sequences or series below, determine if it converges. If it converges, give the limit and a brief justification. If it fails to converge, write that it diverges and give a justification.

$\boxed{\text{5 pts.}}$ (a) $\displaystyle\sum_{j=1}^{\infty} \frac{\pi}{(\ln 8)^j}$

$\boxed{\text{5 pts.}}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\pi}{j - \ln 8}$

$\boxed{\text{5 pts.}}$ (c) $\displaystyle\left\{(-1)^{2n} + \frac{n-1}{n}\right\}_{n=1}^{\infty}$

$\boxed{\text{5 pts.}}$ (d) $\{a_n\}_{n=1}^{\infty}$ where $a_1 = 1$ and $a_{k+1} = 1 + \dfrac{1}{1+a_k}$

This page was once a tree, and once it was blank. Now it is neither. You can make it less blank if you like. Turning it back into a tree is more effort, but also more worthwhile.