MAT 141 Honors Calculus
Exam 2
16 November 2009

Instructor: Joshua Bowman

Name (please print): ____

Instructions:

- **STAY CALM. DON'T PANIC.** ☺
- Please wait to begin the exam until after everyone present has received it.
- The exam consists of six pages (not counting this cover), with six questions. Please check that you have all the pages.
- Read each question carefully, and answer the same way. Partial credit will be given where appropriate.
- No calculators, notes, or textbooks are allowed on this exam.
- You may leave when you have finished.

Question	Points	Score
1	18	
2	18	
3	18	
4	14	
5	18	
6	14	
Total	100	

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work.

Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Signature: ____

1. (3 points each)

   (a) Let a be a point in the domain of a function f that is not an isolated point. Define precisely what it means for f to be continuous at a.

   (b) Let S be a subset of $\mathbb{R}$. Define $\sup S$ and $\inf S$.

   (c) State the Intermediate Value Theorem.

   (d) State the Extreme Value Theorem.

   (e) State the Squeeze Theorem for functions.

   (f) Give the definition of the derivative $f'(a)$ of a function f at a point a in its domain.

2. Find the value of each of the following limits, if it exists; otherwise, write **D.N.E.** (does not exist). (3 points each)

   (a) $\lim_{x \to -1} \frac{x+1}{x^2-1}$

   (b) $\lim_{x \to 0} \frac{x}{\sin x}$

   (c) $\lim_{x \to 0} \frac{x}{\cos x}$

   (d) $\lim_{x \to \infty} \tan \frac{1}{x}$

   (e) $\lim_{x \to \infty} \tan^{-1} x$

   (f) $\lim_{x \to 1} \ln |x-1|$

3. Compute the following derivatives. (3 points each)

(a) $f'(3)$, where $f(x) = 3x^3 - 2x^2 + x - 1$

(b) $g'(2)$, where $g(x) = \tan^{-1} x$

(c) $\dfrac{d}{dx}\big((x + \cos x)e^x\big)$

(d) $\dfrac{d}{dx} \ln(1 + x^2)$

(e) $\dfrac{d}{dx}\left(\dfrac{\sin(x^3)}{1 + e^x}\right)$

(f) $\dfrac{d}{dx} \sin((x + 1)^2(x + 2))$

4. (a) Recall that the hyperbolic sine and cosine functions are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \text{and} \qquad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Show that $\frac{d}{dx} \sinh x = \cosh x$ and $\frac{d}{dx} \cosh x = \sinh x$. (*Note:* unlike the case of the trigonometric functions, the signs of these do *not* change when taking derivatives.) (6 points)

(b) Use part (a) and the relation $\cosh^2 x - \sinh^2 x = 1$ to find the derivative of $\sinh^{-1} x$, the inverse hyperbolic sine. (Use the Inverse Function Rule.) (8 points)

5. Let $p(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial with $a > 0$ and $d < 0$.

(a) Show that $\lim_{x \to \infty} \frac{p(x)}{x^3} = a$. (6 points)

(b) Use part (a) to show that $p(x) > 0$ for some $x > 0$. (6 points)

(c) Use part (b) and the Intermediate Value Theorem to show that $p(x)$ equals zero for some $x > 0$. (6 points)

6. (a) Let f and g be functions defined on all of $\mathbb{R}$. Suppose that f is strictly increasing and g is strictly decreasing. Show that there is at most one point of $\mathbb{R}$ where f and g are equal. Do *not* assume that either function is differentiable. (*Hint:* What happens if you assume that f and g are equal at two distinct points of $\mathbb{R}$?) (8 points)

(b) Give an example of a pair of continuous functions f and g defined on all of $\mathbb{R}$ such that f is strictly increasing, g is strictly decreasing, and f and g are *never* equal. (6 points)

When I heard the learn'd astronomer;
When the proofs, the figures, were ranged in columns before me;
When I was shown the charts and the diagrams, to add, divide, and measure them;
When I, sitting, heard the astronomer, where he lectured with much applause in the lecture-room,
How soon, unaccountable, I became tired and sick;
Till rising and gliding out, I wander'd off by myself,
In the mystical moist night-air, and from time to time,
Look'd up in perfect silence at the stars.

—*Walt Whitman*