MAT 141 Honors Calculus
Exam 1
9 October 2009

Instructor: Joshua Bowman

Name (please print): ______________________________

Instructions:

- **STAY CALM. DON'T PANIC.** ☺
- Please wait to begin the exam until after everyone present has received it.
- The exam consists of five pages (not counting this cover), with six questions. Please check that you have all the pages.
- Read each question carefully, and answer the same way. Partial credit will be given where appropriate.
- No calculators, notes, or textbooks are allowed on this exam.
- You may leave when you have finished.

Question	Points	Score
1	24	
2	18	
3	15	
4	10	
5	18	
6	15	
Total	100	

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work.

Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Signature: ______________________________

1. (3 points each)

   (a) Suppose that A and B are sets. Give the definitions of $A \cup B$ and $A \cap B$.

   (b) What does it mean for an interval to be bounded?

   (c) What does it mean for a sequence to be bounded?

   (d) Give the precise meaning of the phrase, "the sequence $\{a_n\}_{n=0}^{\infty}$ converges to L."

   (e) State the Completeness Axiom.

   (f) State the Bolzano–Weierstrass Theorem.

   (g) State the Comparison Test for positive series.

   (h) State the Alternating Series Test.

2. For each of the following sequences and series, find its limit or state that it diverges. (3 points each)

   (a) $(-1)^n + \dfrac{1}{n}$

   (b) $\dfrac{3n^3 - 1}{2 - n^2 + n^3}$

   (c) $\dfrac{\cos k}{k!}$

   (d) $1 - \dfrac{\pi^2}{2} + \dfrac{\pi^4}{24} - \dfrac{\pi^6}{720} + \cdots + (-1)^k \dfrac{\pi^{2k}}{(2k)!} + \cdots$

   (e) $\displaystyle\sum_{k=0}^{\infty} \left(-\frac{1}{2}\right) \left(-\frac{1}{4}\right)^k$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{3}{2k}$

3. Use induction to prove that

$$3 + 11 + \cdots + (8n - 5) = 4n^2 - n$$

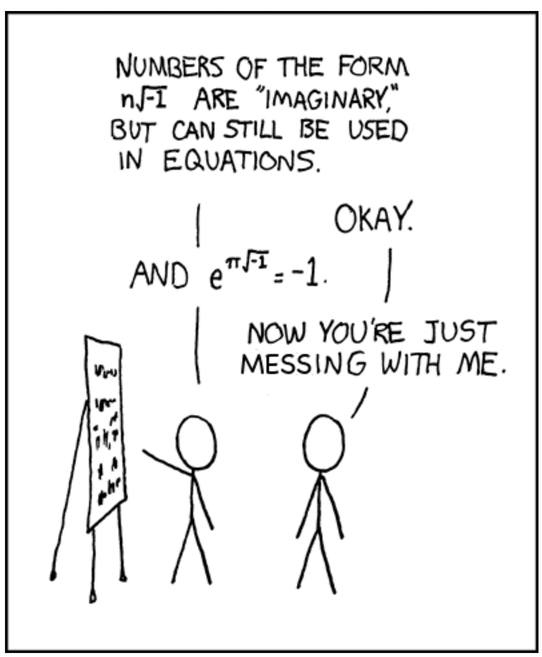
for all integers $n \geq 1$. (15 points)

4. Find all accumulation points of the sequence $z_n = i^n$. (10 points)

5. Suppose that $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ are convergent sequences and that $b_n - a_n$ converges to 0. Show that a_n and b_n have the same limit. (We have used this fact several times in class. Do not just assume this is obvious. Give a clear and correct proof, using any definitions and theorems you find appropriate.) (18 points)

6. (a) Using the series definition of e^x, show that $e^a > 1$ whenever $a > 0$. (8 points)

(b) Show that if $b < 0$, then $0 < e^b < 1$. (*Hint:* Think of b as $-a$ for some positive a and use the key property of exponentials.) (7 points)



comic by Randall Munroe
`http://xkcd.com/179/`