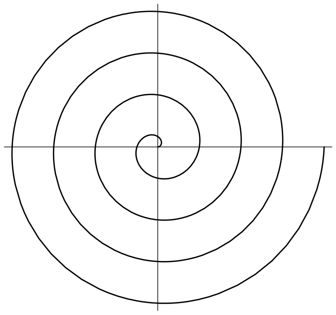
MATH 132 Midterm 2 practice

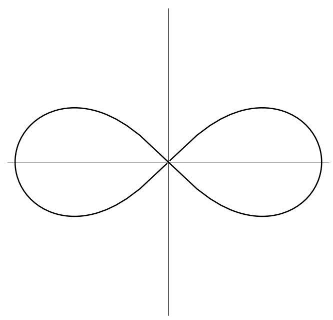
1234 pts 1. There are a bunch of problems about work and volume and stuff on the Spring 2010 exam. So those aren't here. Go do those, OK?

47 pts 2. Match the following polar equations to their graphs. Please write the letter of the graph in the space preceeding the equation. Note that, although each graph is accurate, two different graphs may be drawn at different scales.

____ $r = \cos 2\theta$ ____ $r = \theta \ (\theta > 0)$ ____ $\theta = \frac{\pi}{4}$ ____ $r = 1 + 2\sin\theta$



A



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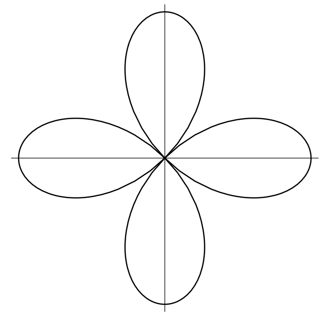

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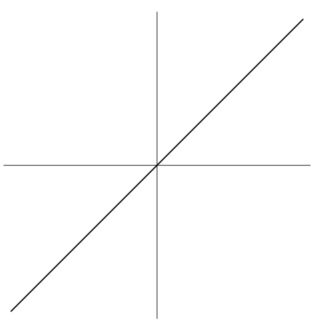

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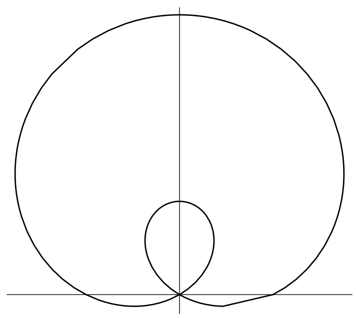

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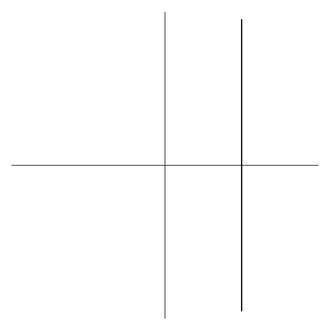

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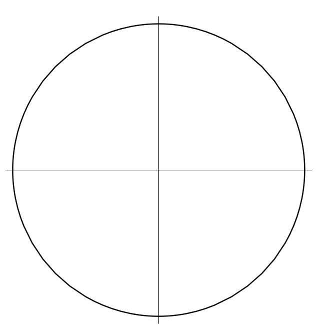

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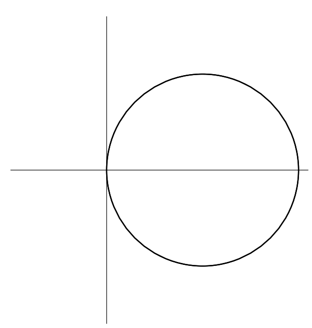
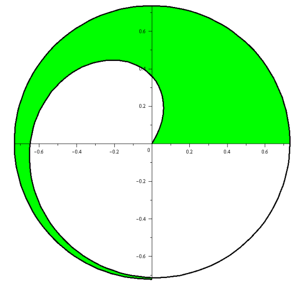


H

47 pts 3. At right is shown the graph of the polar curve



$$r = \frac{\ln\theta}{\sqrt{\theta}} \qquad 1 \le \theta \le \frac{7\pi}{2}$$

Calculate the area of the shaded region.

4. Find the limit of each of the following infinite sequences, if it converges. If the sequence does not converge, say so. Justify your answer.

4 pts (a) $\left\{\dfrac{2^n}{3^{n+1}}\right\}_{n=0}^{\infty} = \dfrac{1}{3}, \dfrac{2}{9}, \dfrac{4}{27}, \ldots$

4 pts (b) $\left\{\dfrac{(n+5)(n+6)(n+12)}{2n^2+24n+36)}\right\}_{n=0}^{\infty} = 10, \dfrac{273}{31}, \dfrac{196}{23}, \dfrac{60}{7}, \ldots$

4 pts (c) $\left\{1+\left(\dfrac{-1}{3}\right)^n\right\}_{n=0}^{\infty} = 1, \dfrac{2}{3}, \dfrac{10}{9}, \dfrac{26}{27}, \ldots$

5. Find the sum of the following infinite series

8 pts (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2\cdot 3^n} = \frac{1}{6} - \frac{1}{18} + \frac{1}{54} - \ldots$

8 pts (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)} = \frac{2}{3} + \frac{1}{4} + \frac{2}{15} + \frac{1}{12} + \ldots$

For each of the inifinite sums below, state whether it converges absolutely, converges (but not absolutely), or diverges. To receive any credit, you must justify your answer fully.

47 pts 6. $\sum_{n=2}^{\infty} \frac{1}{n \ln n} = \frac{1}{2 \ln 2} + \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \ldots$

47 pts 7. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = -1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{2} - \ldots$

47 pts 8. $\sum_{n=0}^{\infty} (-1)^n 2^{-1/n^2} = 1 - \frac{1}{2} + \frac{1}{\sqrt[4]{2}} - \ldots$

47 pts 9. $\sum_{n=0}^{\infty} \frac{e^n}{n!} = 1 + e + \frac{e^2}{2} + \frac{e^3}{6} + \frac{e^4}{24} + \ldots$

47 pts 10. $\sum_{n=0}^{\infty} \frac{n+1}{n^2 - n + 3} = \frac{1}{3} + \frac{2}{3} + \frac{3}{5} + \frac{4}{9} + \ldots$

47 pts 11. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1} = 1 + \frac{x-2}{2} + \frac{(x-2)^2}{3} + \frac{(x-2)^3}{4} + \dots$$

47 pts 12. Find the interval of convergence for the power series corresponding to the Bessel function

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2} = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2306} + \dots$$

47 pts 13. (a) Show that the infinite sum $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 4}{2n+1} = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \dots$ converges.

(b) In fact, the series converges to π. What value of N do we need to use to ensure that $\sum_{n=0}^{N} \frac{(-1)^n \cdot 4}{2n+1}$ is within $\frac{1}{100}$ of π?