MAT 132, Calculus II **SAMPLE FINAL PROBLEMS**

December 20, 2006

The exam will consist in ten problems like the ones below. Each problem will worth between 9 and 12 points.

SHOW ALL WORK TO GET FULL CREDIT; A CORRECT ANSWER WITH INCORRECT JUSTIFICATION WILL NOT GET CREDIT.

1. Evaluate the integrals
   (a) $\int \tan^2(x)\sec^4(x)dx$
   (b) $\int \frac{x^2-1}{x^3+x}dx$
   (c) $\int \sqrt{x}\ln(x)dx$
   (d) $\int x^3\ln(x)dx$
   (e) $\int \tan(10x)dx$ (the removed exercise $\int \frac{\sin(x)}{(1+x^2)}dx$ was beyond the limits of the course)
   (f) $\int x\sin^2(x)\cos(x)dx$

2. A function f defined on $0 \leq x \leq 40$ has the following values

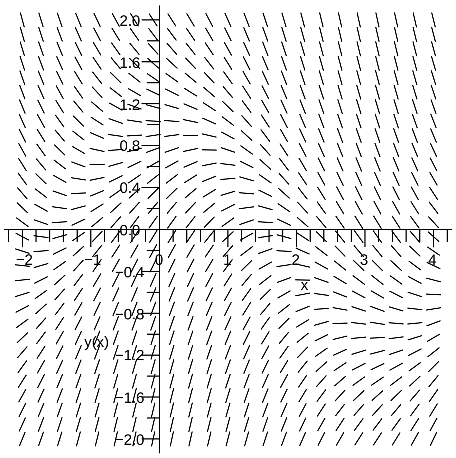
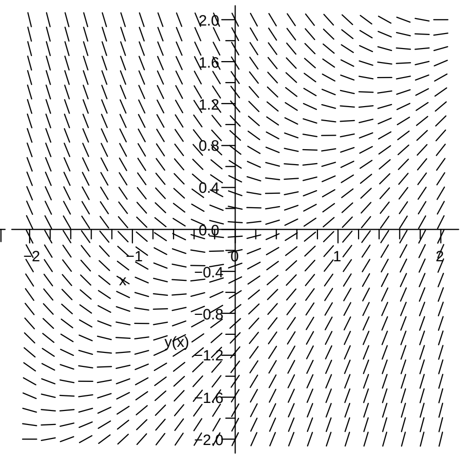
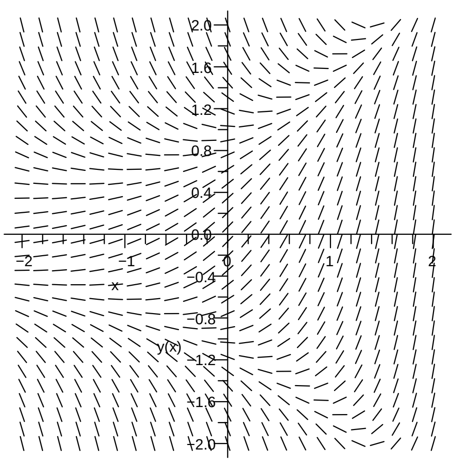
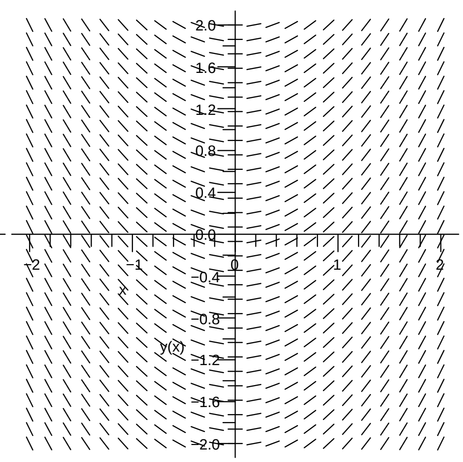
x	$f(x)$
0	2
10	2.5
20	4.5
30	5
40	0

   (a) (6 points) Approximate $\int_0^{40} f(x)\,dx$ using the trapezoid rule with 2 (two) equal subdivisions.
   (b) (6 points) Approximate the integral using Midpoint's rule and with 2 (two) equal subdivisions.

3. Consider the curve defined by the parametric equations $x = t^2$, $y = t^3$.
   (a) Set up, but do not evaluate an integral that represents the arc length of the curve for $10 \leq t \leq 20$.
   (b) Find the arc length of the curve when $1/3 \leq t \leq 1/2$

4. Draw the directions field of the differential equation $\frac{dy}{dx} = y^2$. Use to sketch two solutions curves, one with $y(1) = 1$ and the other $y(0) = 0$

5. Each of the diagrams below depicts the direction fields corresponding to a differential equation of the form $\frac{dy}{dx} = f(x,y)$.


2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2
−1
0
1
2
3
4
x
y(x)

2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2
−1
0
1
2
x
y(x)

2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2
−1
0
1
2
x
y(x)

2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2
−1
0
1
2
x
y(x)

(a) Which one diagrams correspond to the function $f(x,y) = x - y$? (Justify your answer.)

(b) In the diagram you found in a. sketch the solution of the initial value problem , $\frac{dy}{dx} = f(x,y), \quad y(-1) = 0$.

6. Sketch the region bounded by the given curves, and find the volume of the solid of revolution that is formed when the region is revolved around the x-axis.

   (a) $y = \sqrt{x}$, $y = 0$, $x = 1$.

   (b) $y = 1 + e^x$, $y = 0$, $x = 0$, $x = 1$.

7. (a) Sketch the graph of the region determined by the curve $y = \cos(x)$, between $x = 0$ and $x = \frac{\pi}{2}$.

   (b) Compute the area of this region.

   (c) Compute the volume of the solid obtained by revolving this region about the x-axis.

   (d)

8. Suppose that $y(x)$ satisfies the differential equation $y' = \frac{y^2}{x}$

   (a) Give a general solution.

   (b) If $y(x)$ is a solution and $y(1) = 0$, find $y(x)$.

9. Consider the differential equation $\frac{dy}{dx} = y - x^2$

   (a) Determine which one of the following functions is a solution, *explaining your reasoning* (C is a constant).

      i. $y(x) = \cos(Cx)$

      ii. $y(x) = 2 + 2x + Cx^2 + e^x$

      iii. $y(x) = 2 + 2x + x^2 + Ce^x$

   (b) Solve the initial value problem $\frac{dy}{dx} = y - x^2$, $y(0) = 3$.

10. Let $\{a_n\}_{n \leq 1}$ be a sequence with $a_n = \frac{\cos(n)}{2^n}$.

    (a) List the first, third and tenth term of the sequence.

    (b) Determine whether the sequence if convergent and if so, find the limit.

    (c) Determine wether the series $\sum_{n=1}^{\infty} a_n$ is convergent.

11. Write the infinite repeating decimal 0.55555... as a geometric series. Use the formula for the sum of a geometric series to express this as a fraction (ratio of two whole numbers).

12. Write the infinite repeating decimal 0.17171717... as a geometric series. Use the formula for the sum of a geometric series to express this as a fraction (ratio of two whole numbers).

13. Determine if the following series converge or diverge. Justify.

$$(a)\quad \sum_{n=1}^{\infty} \frac{1+n^2 2^{-n}}{n^2} \qquad (b)\quad \sum_{n=1}^{\infty} \frac{2n+1}{n^3+n} \qquad (c)\quad \sum_{n=1}^{\infty} \frac{\cos n}{n^2} \qquad (d)\quad \sum_{n=1}^{\infty} \frac{2^n}{n!} \qquad (d)$$

14. Let $f(x) = \frac{1}{1-x}$.

    (a) Write down a power series which equals $f(x)$ for all $|x-3| < 1$.

    (b) Write down a power series which equals $(x-3)f'(x)$ for all $|x-3| < 1$.

15. (a) Write $f(x) = \frac{1}{2+x^2}$ as a geometric series.

    (b) Use the power series representation of f to find calculate the twentieth derivative of f at $x = 0$.

16. Find exactly for which values of x each of the following power series converges.

$$(a)\quad \sum_{n=1}^{\infty} \frac{n}{e^n}(x-3)^n \qquad (b)\quad \sum_{n=1}^{\infty} \frac{\ln n}{\ln(n+1)} x^n \qquad (c)\quad \sum_{n=1}^{\infty} \frac{3^n}{n^3} x^n$$

17. Find the exact value of $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

18. Evaluate the indefinite integral $\int \frac{\sin(x)}{x}$ as an infinite series

19. (a) Find all the real functions $y(x)$ which satisfy the equation $y'' + y' + 2y = 0$.

    (b) Find all the real solutions passing though the point (0,1). How many of these can you find?

    (c) Find all the real solutions whose derivative passes through the point (0,-1).