Name: ________________________

ID#: ________________________

Test # 2

MAT 127 Spring 2005

Directions: There are 5 questions. You have until 10 PM (90 minutes). For credit, you must show all your work, using the backs of the pages if necessary. You may not use a calculator.

1. _____/20   2. _____/20   3. _____/20   4. _____/20   5. _____/20

Total Score. ____/100

1. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y - y^2.$$

(a) What are the equilibrium solutions?

(b) Rewrite the differential equation as a logistic equation. Find the growth rate k and the carrying capacity K.

(c) If $y(0) = 3$ write the solution to the initial value problem.

2. Consider the sequence whose n^{th} term is $a_n = n2^{-n}$.

   (a) Show that after $n = 1$ the sequence is decreasing.

   (b) Assume the sequence converges and compute its limit.

3. Compute the limits of the following convergent sequences:

   (a) $\left\{\frac{\ln(1+e^n)}{n}\right\}$

   (b) $\left\{\frac{(-5)^n}{n!}\right\}$

4. Consider the recursively defined sequence:

$$a_1 = 1 \qquad a_{n+1} = 6 - a_n \quad n \geq 1.$$

   (a) Does the sequence converge, diverge, or neither? Explain your answer.
   (b) What happens if the first term is $a_1 = 3$? Explain your answer.

5. Model a population of wild rabbits by assuming it obeys the logistic equation. When the population is very small it grows at a rate of $1/4$ its population size.
   (a) Initially there are 100 rabbits. After $4 \ln 4$ months there are 250 rabbits. Compute the carrying capacity of their population.
   (b) When will there be 450 rabbits?