## MAT 126

## Second Midterm

Thursday March 25, 2010

Name: $\qquad$ ID: $\qquad$ Rec: $\qquad$

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 30 | 45 | 45 | 45 | 45 | 40 | 250 |
| Score: |  |  |  |  |  |  |  |

There are 6 problems in this exam, printed on 6 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate clearly what is where if you expect someone to look at it. Books, calculators, extra papers, and discussions with friends are not permitted. Leave all answers in exact form (that is, do not approximate $\pi$, square roots, and so on.)

If you wish to use your psychic abilities to read the proctor's mind for the answers, you may do so. However, remember that he may be deliberately thinking of the wrong answers during the test.

## You must give a correct justification of all answers to receive credit.

You have 90 minutes to complete this exam.
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1. Determine these EASY antiderivatives. You should be able to do these very well. In these problems, no justification is needed. Remember the ' $+\mathrm{C}^{\prime}$.

6 pts
(a) $\int \frac{3}{x} d x$

6 pts
(b) $\int 4 \sin (x) d x$

6 pts
(c) $\int e^{3 x} d x$
(d) $\int \frac{3}{t^{2}+1} d t$

6 pts
(e) $\int \frac{1}{\sqrt{1-u^{2}}} d u$

Id:
2. In this question we tell you which method we suggest you use. Use the back of the previous page if you need more space.
(a) Suggested method: substitution $\int \frac{z}{1+z^{2}} d z$
(b) Suggested method: substitution $\int \frac{e^{\sqrt{y+2}}}{\sqrt{y+2}} d y$
(c) Suggested method: substitution $\int \frac{\ln (x)}{x} d x$

Id:
3. In this question we tell you which method we suggest you use. Use the back of the previous page if you need more space.
(a) Suggested method: integration by parts $\int x^{5} \ln (x) d x$
(b) Suggested method: integration by parts $\int x e^{2 x} d x$
(c) Suggested method: integration by parts $\int \sin (x) e^{2 x} d x$
4. Determine the following antiderivatives. Use the back of the previous page if you need more space.
(a) $\int \sin ^{3}(x) d x$

15 pts
(b) $\int \frac{1}{\sec (4 x)} d x$

15 pts
(c) $\int \frac{1}{x^{2} \sqrt{x^{2}-1}} d x$
5. Evaluate these definite integrals. Use the back of the previous page if you need more space.

15 pts
(a) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1-x^{2}} d x$

15 pts
(b) $\int_{-100}^{100} \frac{\sin ^{21}(x)}{1+e^{x^{2}}} d x$

15 pts
(c) $\int_{0}^{1} \frac{x}{\sqrt{4-x^{2}}} d x$
6. Since $\int_{0}^{1} \frac{1}{1+x^{2}} d x=\arctan (1)=\frac{\pi}{4}$, evaluating the integral $\int_{0}^{1} \frac{4}{1+x^{2}} d x$ gives $\pi$.

20 pts
(a) Use Simpson's rule with 2 intervals to estimate $\int_{0}^{1} \frac{4}{1+x^{2}} d x$.
(b) How many intervals are needed to estimate $\int_{0}^{1} \frac{4}{1+x^{2}} d x=\pi$ within .0001 using the trapezoid rule? ${ }^{1}$
${ }^{1}$ Use the following estimate for $E_{T}$ using $n$ intervals: If $\left|f^{\prime \prime}(x)\right| \leq K$ then $E_{T} \leq K \frac{(b-a)^{3}}{12 n^{2}}$. If $f(x)=\frac{1}{1+x^{2}}$, then $f^{\prime \prime}(x)=\frac{6 x^{2}-2}{\left(1+x^{2}\right)^{3}}$

