1. Let

$$f(x) = \int_{1}^{1/x} \arctan(t) \, dt$$

Find  $f'\left(\frac{1}{\sqrt{2}}\right)$ .

2. A lemur rancher needs to invest in some high-tech lemur grooming machines. She determines that the machines will depreciate at a rate f(t), and the cost of keeping them in top running condition is given by another function g(t), where t is the time that the machines have been running.

The cost of keeping the machines around (instead of replacing them with new ones) is given by

$$C(t) = \frac{1}{t} \int_0^t \left( f(t) + g(t) \right) \, dt$$

Show the critical points of C(t) occur when C(t) = f(t) + g(t).