Sample Second Midterm Exam MAT 118

Here are some problems you can use to prepare yourself for the exam. Note that this is not an exhaustive set of problems: just because something is here doesn't mean it will be on the exam, and there may be material on the exam not represented here. You should not need a calculator to do any of these problems, but you are welcome to use one anyway.

The exam covers material since the last exam, namely chapters 9, 10 and part of 11 out of the texbook.

The exam will be held on Wednesday, November 28, during class time (9:35 AM). Do not forget to bring your student ID card or another photo ID like a driver's license.

1. If I tell you that the 29th Fibonacci number F_{29} is 514,229, and F_{31} =1,346,269, can you use that information to find F_{30}? How about F_{28}?

2. (a) Find a triangle with one side having length 1 that is its own gnomon. You can identify the triangle either by giving its angles or the length of the other two sides, as you like.

   (b) Is there more than one answer to the first part of this question (that is, is there a different triangle which is its own gnomon?) Please justify your answer fully. (An answer like "I guess not, because I couldn't find it" is not sufficient.)

3. The base of a triangle has length 5", and the base of the another is 10". If the two triangles are similar, and the area of the first is 13 square inches, what is the area of the second?

4. Suppose β is a number such that $\beta^2 = \beta + 3$. Find two whole numbers a and b so that

$$\beta^5 = a\beta + b.$$

(It is not at all necessary to know that $\beta = (1+\sqrt{13})/2 \approx 2.3027756$ to do this problem, nor is a calculator needed.)

5. A nuclear power plant produces 12 pounds of radioactive waste every month, which must be stored in a special tank which can hold 500 pounds of the glowing goo. On January 1 2006, there were 25 pounds of waste in the tank (I guess it came with one pound for good luck). Let P_N represent the amount of radioactive waste in the tank after N months.

   (a) Write an expression (either explicit or recursive) for P_N.

   (b) When will the tank be full?

6. In 2000, there were 100,000 cases of equine flu reported. Each year, the number of cases decreases by 20%. Let E_n be the number of new cases of equine flu reported in the year $2000 + n$.

   (a) Write an expression (either explicit or recursive) for E_n.

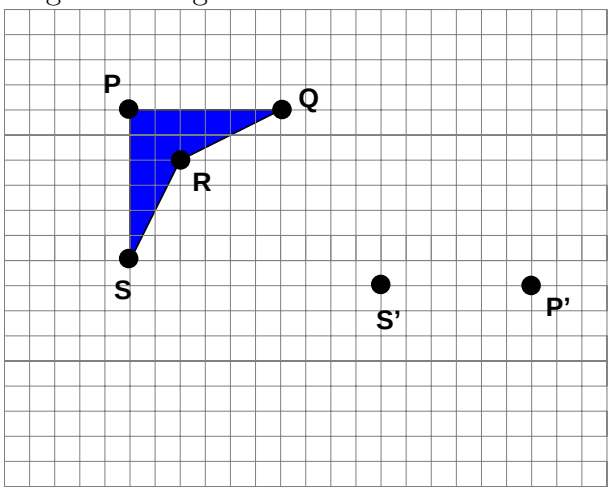
   (b) Assuming the trend continues, how many cases of equine flu should be expected in 2015?

7. A population of rabbits on an island covered with delicious green grass and no predators grows according to the logistic model

$$P_{n+1} = 3P_n(1 - P_n),$$

where P_n represents the number of rabbits after n years as a fraction of the island's carrying capacity. If $P_0 = 1/3$, find the population after 5 years.

8. In the figure below, a glide reflection takes the point marked **P** to the point marked **P'** and the point marked **S** to the point marked **S'**. On the figure, indicate the axis of the reflection, and the image of the figure.



9. How many distict symmetries does a regular hexagon have?