Sample First Midterm Exam MAT 118

Here are some problems you can use to prepare yourself for the exam. Note that this is not an exhaustive set of problems: just because something is here doesn't mean it will be on the exam, and there may be material on the exam not represented here. You should not need a calculator to do any of these problems, but you are welcome to use one anyway.

The exam will be held on Friday, October 26, during class time (9:35 AM). Do not forget to bring your student ID card or another photo ID like a driver's license.

1. The seven members of a spelunking club want to choose one member to go first in exploring a dangerous cave. There are four volunteers: Abelard, Bobo, Cecilia, and Duncan. The votes of the club members are summarized in the table at right.

volunteer	preferences						
Abelard	1	2	4	3	2	3	3
Bobo	3	1	3	4	3	2	2
Cecelia	2	3	1	1	4	4	4
Duncan	4	4	2	2	1	1	1

   (a) Which volunteer would win in a plurality vote?

   (b) Which volunteer would win in an Instant Runoff election? (show your work)

   (c) Which volunteer would win the Borda count? (show your work)

   (d) Which volunteer would win a pairwise comparison contest? (show your work)

   (e) Which volunteer, if any, is the Condorcet winner? (show your work).

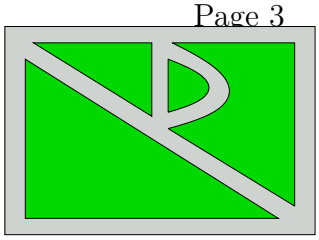
2. Four people hold shares in a corporation, with the number of votes proportional to the number of shares held. To carry a motion, 10 or more votes are needed. A has 6 shares, B has 5 shares, C has 4 shares, and D has 2 shares (so the system is written as [10: 6, 5, 4, 2]).

   (a) What is the Banzhaf power of each player?

   (b) What is the Shapely-Shubik power of each player? There are 24 sequential coalitions, listed below.

(A, B, C, D)	(B, A, C, D)	(C, A, B, D)	(D, A, B, C)
(A, B, D, C)	(B, A, D, C)	(C, A, D, B)	(D, A, C, B)
(A, C, B, D)	(B, C, A, D)	(C, B, A, D)	(D, B, A, C)
(A, C, D, B)	(B, C, D, A)	(C, B, D, A)	(D, B, C, A)
(A, D, B, C)	(B, D, A, C)	(C, D, A, B)	(D, C, A, B)
(A, D, C, B)	(B, D, C, A)	(C, D, B, A)	(D, C, B, A)

3. The sidewalks of a park are shown in the figure on the right (they go around the outside, across the middle, and make the D-shape). A city worker needs to sweep the walks, and wants to cover each one exactly once, without retracing his steps or walking on the grass.



(a) Draw the graph corresponding to this situation.

(b) Is it possible for the worker cover each part of the walk exactly once, ending up where he started? (That is, does the graph have an Eulerian circuit?) Justify your answer.

(c) Is it possible for the worker cover each part of the walk exactly once if he doesn't need to end up where he started? (That is, does the graph have an Eulerian path?) Justify your answer.

4. A person starting in Wichita must visit Kansas City, Omaha, and St. Louis (in any order), then return home to Wichita. You don't need to know that Omaha is north of Wichita, Kansas City is northeast, and St. Louis is east, but I'll tell you anyway. Approximate road mileages between the various cities are given below.

	Kansas City	**Omaha**	**St. Louis**	**Wichita**
Kansas City	—	220	225	280
Omaha	220	—	310	300
St. Louis	225	310	—	500
Wichita	280	300	500	—

(a) Draw a weighted graph which corresponds to the situation.

(b) Use the nearest neighbor algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit? (Write your answers in the spaces below.

Wichita, _____, _____, _____, Wichita. Distance: _____

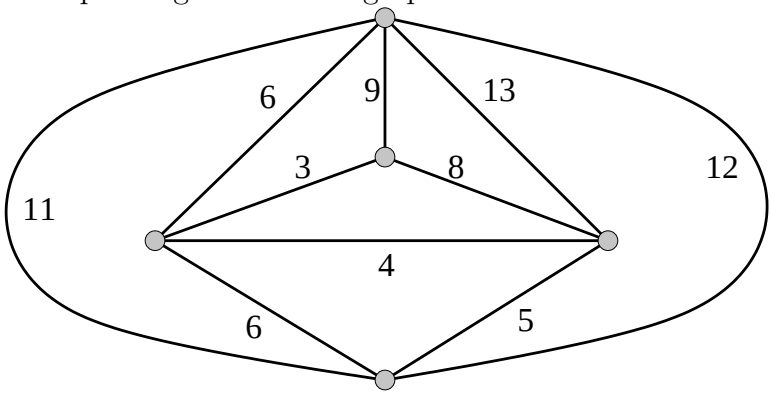
(c) Use the greedy algorithm to find an approximate solution to the traveling salesman problem, making a circuit starting at Wichita. What is the length of this circuit?(Write your answers in the spaces below.

Wichita, _____, _____, _____, Wichita. Distance: _____

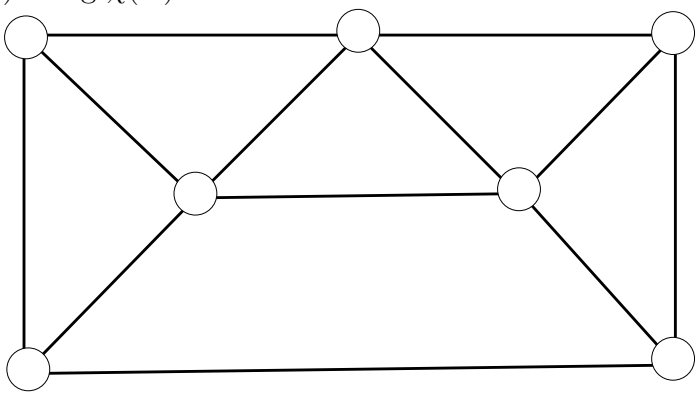
(d) Write the itinerary of the *shortest possible* solution to the traveling salesman problem in ths case.

Wichita, _____, _____, _____, Wichita. Distance: _____

5. Find the minimal spanning tree for the graph below.



6. For the graph below, what is $\chi(G)$? Give a coloring of the graph (you can use numbers instead of colors) using $\chi(G)$ colors.



Explain why this is the minimum number of colors possible.