MATH 118 Second Midterm

November 28, 2007

Name: ______________________ **ID:** __________________

Recitation on: **Mon** **Tues** **Wed** **Thurs**

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	12	8	12	62
Score:							

There are 6 problems in this exam, printed on 3 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **You may use a calculator, but books, extra papers, and discussions with friends are not permitted.** Feel free to confer with Leonhard Euler's ghost if you wish, although make sure his spectral form doesn't frighten your neighbors.

You have an hour to do this exam. There are some things you might or might not find useful on the next page.

Stuff from MATH 118 that you know but don't need to memorize

- Two figures are said to be similar if one is a scaled version of the other. Corresponding parts of similar figures have proportional lengths; corresponding angles have the same measure.

- The gnomon of a shape X is another shape Y such that when the two are joined appropriately, the result is similar to X.

- The Fibonacci numbers F_n satisfy $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ (for $n > 2$). The first few Fibonacci numbers are $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$

- The Golden Ratio is the number $\phi = \dfrac{\sqrt{5}+1}{2} \approx 1.618$. It satisfies the relationship

$$\phi^2 = \phi + 1$$

  and is closely related to the Fibonacci numbers, since $\phi^n = F_n\phi + F_{n-1}$.

- Linear growth occurs when a constant amount is added or subtracted each time period. Such models are of the form $P_n = P_0 + n \cdot d$, where P_0 is the initial amount and d is the amount added (or subtracted, if it is negative).

- An arithmetic sequence is one where the terms grow linearly. The sum of all the terms of an arithmetic sequence can be calculated using the formula

$$A_0 + A_1 + A_2 + \ldots + A_{n-1} = n\frac{(A_0 + A_{n-1})}{2}$$

- Exponential growth occurs when the amount is multiplied by a constant amount each time period. In this case, we have $P_n = P_0 \cdot r^n$. If $r > 1$, we have growth, and if $0 < r < 1$, the amount decreases and we have exponential decay.

- A geometric sequence is one where the terms grow exponentially. The sum of all the terms of a geometric sequence is given by formula

$$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

- A logistic model applies when there are limited resources. Typically we express the population p_n as a fraction of the total possible, so it is a number between 0 and 1. In this case, the logistic model looks like $p_{n+1} = r \cdot p_n \cdot (1 - p_n)$

- The symmetries of a figure are described in terms of rigid motions: translations, rotations, reflections, or glide reflections. Every figure has at least one symmetry, the identity symmetry (or rotation by a full turn).

Name: ______________________ **Id:** ______________________

1. $\boxed{\text{10 points}}$ A rectangle is its own gnomon. If the short side is of length 4, how long is the other side? (Justify your answer at least a little bit).

2. $\boxed{\text{10 points}}$ If $F_{12} = 144$ and $F_{13} = 233$, what is the 15th Fibonacci number F_{15}? (Justify your answer somewhat.)

3. $\boxed{\text{10 points}}$ Without using a calculator, simplify

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\phi}}}}$$

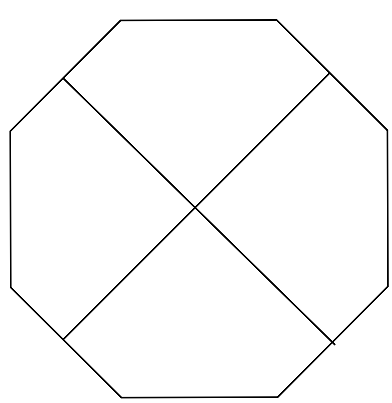
as much as possible. (ϕ is the golden ratio.) Justify your answer.

4. $\boxed{\text{12 points}}$ A jar has 45 cents in it at 8 AM on November 1. Each day at 5 PM, 15 cents are added to the jar.

   (a) Write an equation (either recursive or explict) that describes how much money is in the jar at 6pm on the nth day.

   (b) How many days will it be until the jar has at least \$10 in it?

5. $\boxed{\text{8 points}}$ Describe the symmetries of the figure below (how many are there, what type).



6. $\boxed{\text{12 points}}$ Each of the graphs below describes a population which grows according to a linear, exponential, or logistic model. For each graph, determine which model applies and write “linear”, “exponential”, “logistic”, or “none” next to it.