MATH 118 Second Midterm

October 26, 2007

Name: _____ **ID:** _____ **Rec:** ____

Question:	1	2	3	4	5	Total
Points:	12	12	10	10	12	56
Score:						

There are 5 problems in this exam, printed on 4 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate **clearly** what is where if you expect someone to look at it. **You may use a calculator, but books, extra papers, and discussions with friends are not permitted.** You are free to use a time machine to travel ahead in time and check your answers, provided you also let me use it to go back in time and change the questions.

You have an hour to do this exam. There are some things you might or might not find useful on the back of this page.

Stuff from MATH 118 that you know but don't need to memorize

- The winner of a plurality election receives the most first-place votes.
- In the Borda count, points given based on first choice, second choice, etc.
- In Instant Runoff (or plurality with elimination), candidates with the least votes are repeatedly removed.
- A candidate who wins all head-to-head competitions is the Condorcet winner.
- The pairwise comparison method (Copeland's Method) compares each candidate in head-to-head competitions.
- In a weighted voting system, a player who has no power is called a "dummy" and a player with all the power is a "dictator". A player who must vote yes for a motion to carry is said to have "veto power".
- The Banzhaf power index depends on who is the critical player in the (unordered) winning coalitions.
- The Shapely-Shubik power index depends on who is the pivotal player in sequential (ordered) voting.
- An Eulerian Circuit (or path) is one which contains each edge exactly once.
- A Hamiltonian Circuit (or path) contains each vertex exactly once.
- The nearest neighbor algorithm for solving the traveling salesman problem builds a Hamiltonian circuit by choosing the cheapest edge at each stage.
- The greedy algorithm (also called cheapest link) for solving the traveling salesman problem takes the cheapest edges (not in order), as long as they will eventually form a circuit.
- A spanning tree in a graph is a tree using some of the edges of the original graph to connect all of the vertices.
- A Steiner point in a triangle makes a 120-degree angle with each of the three vertices.
- A coloring of a graph assigns colors to its vertices so that no two vertices connected by an edge are the same color.
- The chromatic number of a graph G is denoted $\chi(G)$ and is the minimum number of colors needed to color G.

Name: _____ **Id:** _____

1. [12 points] Seven kids are trying to decide which one of four games to play. The choices are Tag, Wallball, Baseball, and Ultimate Frisbee. The preferences of each child are given below, with the most favorite first.

Anna likes Tag, Wallball, Baseball, then Frisbee.
Bob likes Baseball, Tag, Wallball, then Frisbee.
Calvin likes Wallball, Frisbee, Baseball, then Tag.
Diane likes Wallball, Frisbee, Tag, then Baseball.
Egbert likes Frisbee, Tag, Baseball then Wallball.
Francis likes Frisbee, Baseball, Tag, then Wallball.
Gina likes Frisbee, Baseball, Tag, then Wallball.

	preferences						
	A.	B.	C.	D.	E.	F.	G.
1st	T	B	W	W	F	F	F
2nd	W	T	F	F	T	B	B
3rd	B	W	B	T	B	T	T
4th	F	F	T	B	W	W	W

(a) Which game would win in a plurality vote?

(b) Which game would win an Instant Runoff?

(c) Which game would win the Borda count? (show your work)

(d) Which game would win a pairwise comparison? (show your your work).

2. [12 points] Four people hold shares in a corporation, with the number of votes proportional to the number of shares held. To carry a motion, 12 or more votes are needed. A has 6 shares, B has 5 shares, C has 4 shares, and D has 2 shares (so the system is written as [12: 6, 5, 4, 2]).

   (a) What is the Banzhaf power of each player?

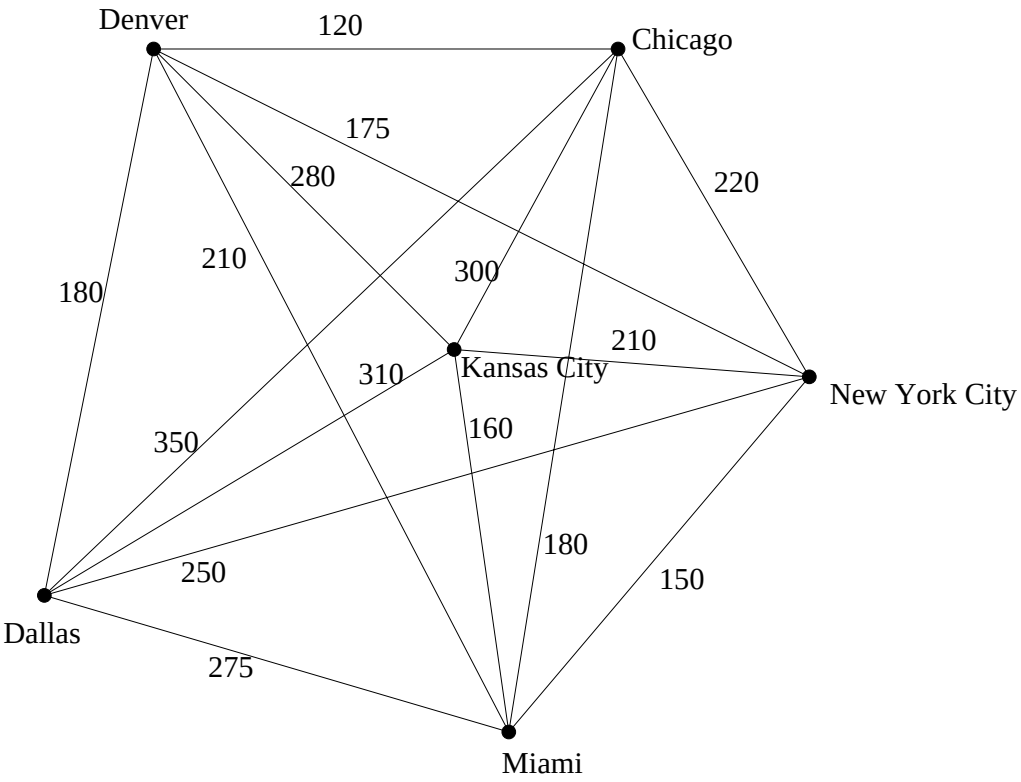
   (b) Identify below any player who is a dummy, a dictator, or has veto power. Write "none" if there are none.

   dictator: dummy: veto power:

   (c) What is the Shapely-Shubik power of each player? There are 24 sequential coalitions, listed below.

(A, B, C, D)	(B, A, C, D)	(C, A, B, D)	(D, A, B, C)
(A, B, D, C)	(B, A, D, C)	(C, A, D, B)	(D, A, C, B)
(A, C, B, D)	(B, C, A, D)	(C, B, A, D)	(D, B, A, C)
(A, C, D, B)	(B, C, D, A)	(C, B, D, A)	(D, B, C, A)
(A, D, B, C)	(B, D, A, C)	(C, D, A, B)	(D, C, A, B)
(A, D, C, B)	(B, D, C, A)	(C, D, B, A)	(D, C, B, A)

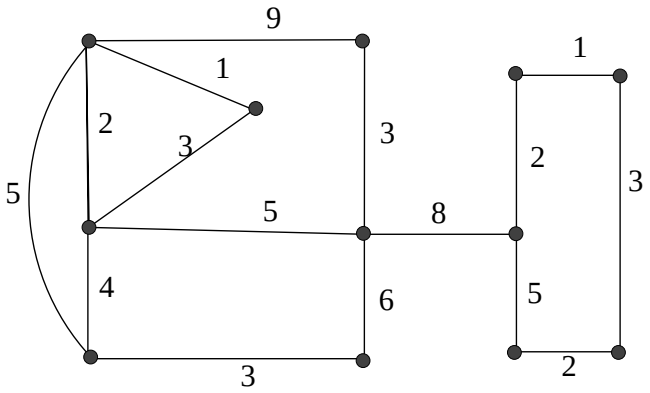
3. [10 points] You need to visit each of the cities Chicago, Denver, Dallas, Miami, and Kansas City, starting from and returning to New York. The airfares are listed on the graph. Use the greedy algorithm (also called "Cheapest Link") to plan your trip, attempting to minimize the cost. (That is, try to find a cheap Hamiltonian circuit.)



List the order of the cities in the spaces below. You don't need to give the cost.

NYC, _____, _____, _____, _____, _____, NYC.

4. [10 points] Find the minimum spanning tree for the graph shown below.



5. [12 points] Answer each of the questions below. No justification is necessary.

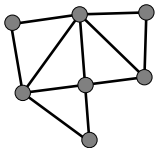
(a) True or False: Every graph can be colored with no more than four colors.

(b) True or False: The complete graph on 30 vertices, K_{30}, has $30!$ edges in it.

(c) Simplify the following expression: $\dfrac{20! + 18!}{19!}$

(d) True or False: Every election has a Condorcet winner.

(e) True or False: The graph below has an Eulerian circuit.



(f) True or False: The graph which is a circuit with 12 vertices and 12 edges has a chromatic number of 2.