Formulas For Final Exam

$$\cos^2 x - \sin^2 x = \cos 2x$$
$$e = \lim_{x\to 0}(1+x)^{1/x}$$
$$e^x = \lim_{n\to\infty}(1+\frac{x}{n})^n$$

Euler's method If $y' = f(x, y)$, then

$$y_0 = y(x_0),$$
$$x_{n+1} = x_n + dx,$$
$$y_{n+1} = y_n + f(x_n, y_n)dx.$$

Improved Euler's method

$$y_0 = y(x_0),$$
$$x_{n+1} = x_n + dx,$$
$$z_{n+1} = y_n + f(x_n, y_n)dx,$$
$$y_{n+1} = y_n + \frac{f(x_n, y_n) + f(x_{n+1}, z_{n+1})}{2}dx,$$

Little-o As $x \to a$, we have:

1. $o(g(x)) = \pm o(g(x)) = o(g(x))$.
2. $o(cg(x)) = \pm o(g(x)), \quad c \neq 0$.
3. $f(x) \cdot o(g(x)) = o(f(x)g(x))$.
4. $o(o(g(x))) = o(g(x))$.
5. $(1 + g(x))^{-1} = 1 - g(x) + o(g(x))$ if $g(x) \to 0$ as $x \to a$.

Fourier coefficients

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx,$$
$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx, \quad k \geq 1,$$
$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx, \quad k \geq 1.$$

The Fourier series is given by

$$a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx).$$

Binomeal coefficients are given by

$$\binom{m}{0} = 1, \quad \binom{m}{k} = \frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}, \quad k \geq 1.$$