

Math 260

September 2, 2008

1. Find the sum of the numbers from 1 to 100.
2. Find the sum of the numbers $1 + 3 + 9 + 27 + \dots + 3^6$.
3. There are 100 light bulbs lined up in a row in a room. Each bulb has its own switch and is currently switched off. The room has an entry door and an exit door. There are 100 students lined up outside the entry door. Each bulb is numbered consecutively from 1 to 100. Student number 1 enters the room, switches on every bulb, and exits. Student number 2 switches off bulbs 2, 4, 6, \dots . Student number 3 enters and flips the switch on every 3rd bulb. This continues until all 100 students have flipped the switches.
 - (a) In the end, is bulb number 64 on or off?
 - (b) In the end, how many bulbs are on?
4. Show that
$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$
5. How many zeros are at the end of $100!$?
6. A box contains two coins. One coin is heads on both sides and the other is heads on one side and tails on the other. One coin is selected from the box at random and the face of one side is observed. If the face is heads what is the probability that the other side is heads?
7. You are a cook in a remote area with no clocks or other way of keeping time other than a 4 minute hourglass and a 7 minute hourglass. You do have a stove however with water in a pot already boiling. Somebody asks you for a 9 minute egg, and you know this person is a perfectionist and will be able to tell if you undercook or overcook the eggs by even a few seconds. What is the least amount of time it will take to prepare the egg?
8. You have nine pearls, eight are real and one is fake. All the real ones weigh the same and the fake weighs less than the real ones. Using a balance scale twice how can you weed out the fake one?
9. It is impossible to draw this figure without taking the pen off the paper, redrawing any lines, or other trickery. Explain why it is impossible.