Math 200 Lecture 1, Fall 2008 Extra Credit

Remember to justify (prove) all answers.

INCIDENCE AXIOM:
(1) For any two distinct points, there is a unique line that contains these two points.
(2) Every line contains at least two distinct points.
(3) For any line, there exists a point not on this line.

PARALLEL AXIOM: For any line l and a point P not on l, there exists a unique line containing P and parallel to l.

RULER AXIOM: Let l be any line. Then there is a one-to-one correspondence $f : l \to \mathbb{R}$ such that, for any two points A,B on l, $|AB| = |f(A) - f(B)|$.

Define l and m to be the subsets of $\mathbb{R}^2$ defined by $l = \{(x, 0) \in \mathbb{R}^2 | 0 < x < 1\}$ and $m = \{(0, y) \in \mathbb{R}^2 | 0 < y < 1\}$. Define the entire plane to be $l \cup m$. We let the distance between the points $P = (x_P, y_P)$ and $Q = (x_Q, y_Q)$ be given by $|PQ| = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$.

1. (8 pts) Is the incidence axiom satisfied?

   No. There is no line that contains both $(1/2, 0)$ and $(0, 1/2)$.

2. (8 pts) Is the parallel axiom satisfied?

   Yes. Given l and a point P not on l, P must be on m, which is the unique line parallel to l. The proof is similar if m is given along with a point not on m.

3. (12 pts) Is the ruler axiom satisfied?

   No. Suppose $f : l \to \mathbb{R}$ is a coordinate system. Let $x = f(1/2, 0)$. The distance from $(1/2, 0)$ to $f^{-1}(x + 2)$ must be $|x - (x + 2)| = 2$. Yet any two points on l have a distance less than 1 between them.