HW - 12 - 1

6.4; We proved, in Thm 6.5, That $m\angle A + m\angle B + m\angle C = \pi$. ①

We, also, know that as $\angle BCD$ is a straight angle,

$m\angle BCD = \pi$.

And we know that $m\angle ACD + m\angle ACB = m\angle BCD = \pi$ ②

①, ② $\Rightarrow$

$$\underline{m\angle C} + m\angle ACD = m\angle A + m\angle B + \underline{m\angle C}$$

$$\rightarrow m\angle ACD = m\angle A + m\angle B \qquad \blacksquare$$

HW 6.6; By definition of points on the opposite side of a line,

if $\overline{BD} \cap \overline{AC} \neq \emptyset$ (That is they meet) $\rightarrow \begin{cases} \overleftrightarrow{BD} \cap \overline{AC} \neq \emptyset & ① \\ \overline{BD} \cap \overleftrightarrow{AC} \neq \emptyset & ② \end{cases}$.

So $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} \neq \emptyset$. By uniqueness of point of intersection,

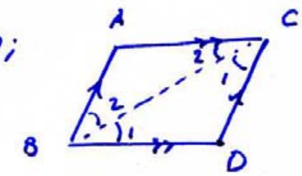
$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \overline{AC} \cap \overleftrightarrow{BD} = \overleftrightarrow{AC} \cap \overline{BD} = \{D\}$.

$\rightarrow \begin{cases} D \in \overline{AC} \\ D \in \overline{BD} \end{cases} \Rightarrow \overline{AC} \cap \overline{BD} \neq \emptyset$. That is they meet at D.

$\blacksquare$

HW-12-2

6.10;



$$*\left\{\begin{array}{l}\overline{AC}\parallel\overline{BD}\Rightarrow m\angle C_2 = m\angle B_1.\\ \overline{AB}\parallel\overline{CD}\Rightarrow m\angle C_1 = m\angle B_2\\ |\overline{BC}| = |\overline{BC}|\ \text{shared side}\end{array}\right\}\xrightarrow{ASA.}\ \overset{\triangle}{ABC}\cong\overset{\triangle}{BCD}$$

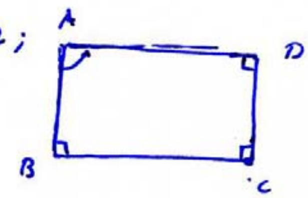
$\Rightarrow m\angle D = m\angle A.$ ✓

$* \rightarrow m\angle C_1 + m\angle C_2 = m\angle B_1 + m\angle B_2 \rightarrow m\angle C = m\angle B.$ ✓

$|\overline{CD}| = |\overline{AB}|$ ✓

$|\overline{AC}| = |\overline{BD}|$ ✓ ■

6.12;



$m\angle A = m\angle B = m\angle D = m\angle C.$

$\rightarrow$ as $m\angle A = m\angle B \rightarrow \overleftrightarrow{AD}\parallel\overleftrightarrow{BC}$

(alternate angles).

$\rightarrow$ as $m\angle B = m\angle C \rightarrow \overleftrightarrow{AB}\parallel\overleftrightarrow{CD}$

By definition of parallelogram, $\Diamond ABCD$ is a prllgrm. ■

7.1;

$$\frac{|A'B'|}{|AB|} = \frac{|B'C'|}{|BC|} = \frac{|A'C'|}{|AC|} = k_1.$$

$$\frac{|A''B''|}{|A'B'|} = \frac{|B''C''|}{|B'C'|} = \frac{|A''C''|}{|A'C'|} = k_2. \longrightarrow \text{multiplying the equalities:}$$

$$\frac{|A''B''|}{|A'B'|}\cdot\frac{|A'B'|}{|AB|} = \frac{|A''B''|}{|AB|} = \frac{|B''C''|}{|BC|} = \frac{|A''C''|}{|AC|} = k_1 k_2.$$

■

HW - 12 - 3;

7-3; By construction, $\left\{\begin{array}{l} C_iD_i \parallel AB' \quad (*) \\ C_{i+1}D_{i+1} \parallel AB' \end{array}\right. \Rightarrow C_iD_i \parallel C_{i+1}D_{i+1}$

$$\rightarrow m\angle D_iC_iC_{i+1} = m\angle D_{i+1}C_{i+1}C_{i+2} \quad (1)$$

$\cancel{C_2B_2 \parallel C_3B_3 .\ \angle B_1B_2D_1 =}$

$\cancel{C_iD_i \parallel C_{i+1}D}$ $C_iB_i \parallel C_{i+1}D_{i+1} \rightarrow m\angle B_{i-1}B_iC_i = m\angle B_iB_{i+1}C_{i+1}$

By $*$, $m\angle C_iD_iC_{i+1} = m\angle B_{i-1}B_iC_i$

$m\angle C_{i+1}D_{i+1}C_{i+2} = m\angle B_iB_{i+1}C_{i+1}$. $\Rightarrow$

$$m\angle C_iD_iC_{i+1} = m\angle C_{i+1}D_{i+1}C_{i+2} . \quad (2)$$

By construction; $|B_iB_{i+1}| = |B_{i+1}B_{i+2}|$.

As $\diamond B_iC_iD_iB_{i+1}$ is a parallelogram (quadrilateral with parallel sides) $|C_iD_i| = |B_iB_{i+1}|$.

$|C_{i+1}D_{i+1}| = |B_{i+1}B_{i+2}|$ $\Rightarrow$.

$$|C_iD_i| = |C_{i+1}D_{i+1}| \quad (3) .$$

$$1, 2, 3 \xRightarrow{ASA} \triangle C_iD_iC_{i+1} \cong \triangle C_{i+1}D_{i+1}D_{i+2} \quad \blacksquare$$

7-4; If $\frac{|AB'|}{|AB|} = \frac{1}{m} \rightarrow \frac{|AB|}{|AB'|} = m$, that is by interchanging AB and AB' in 7.2, $B'C' \parallel BC$. $\blacksquare$